Instrumental Systematics on Lensing Reconstruction and primordial CMB B-mode Diagnostics

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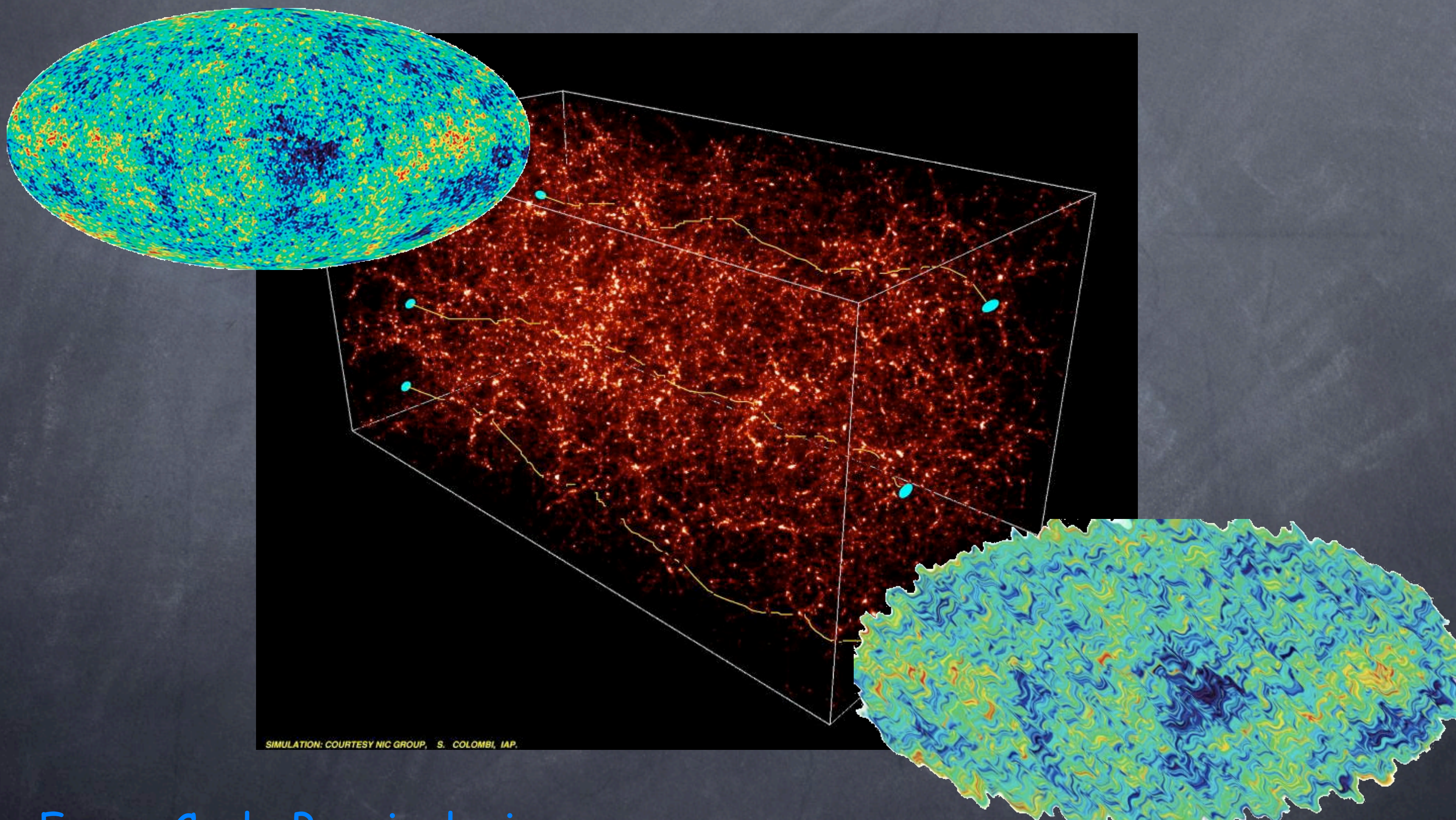
Outline

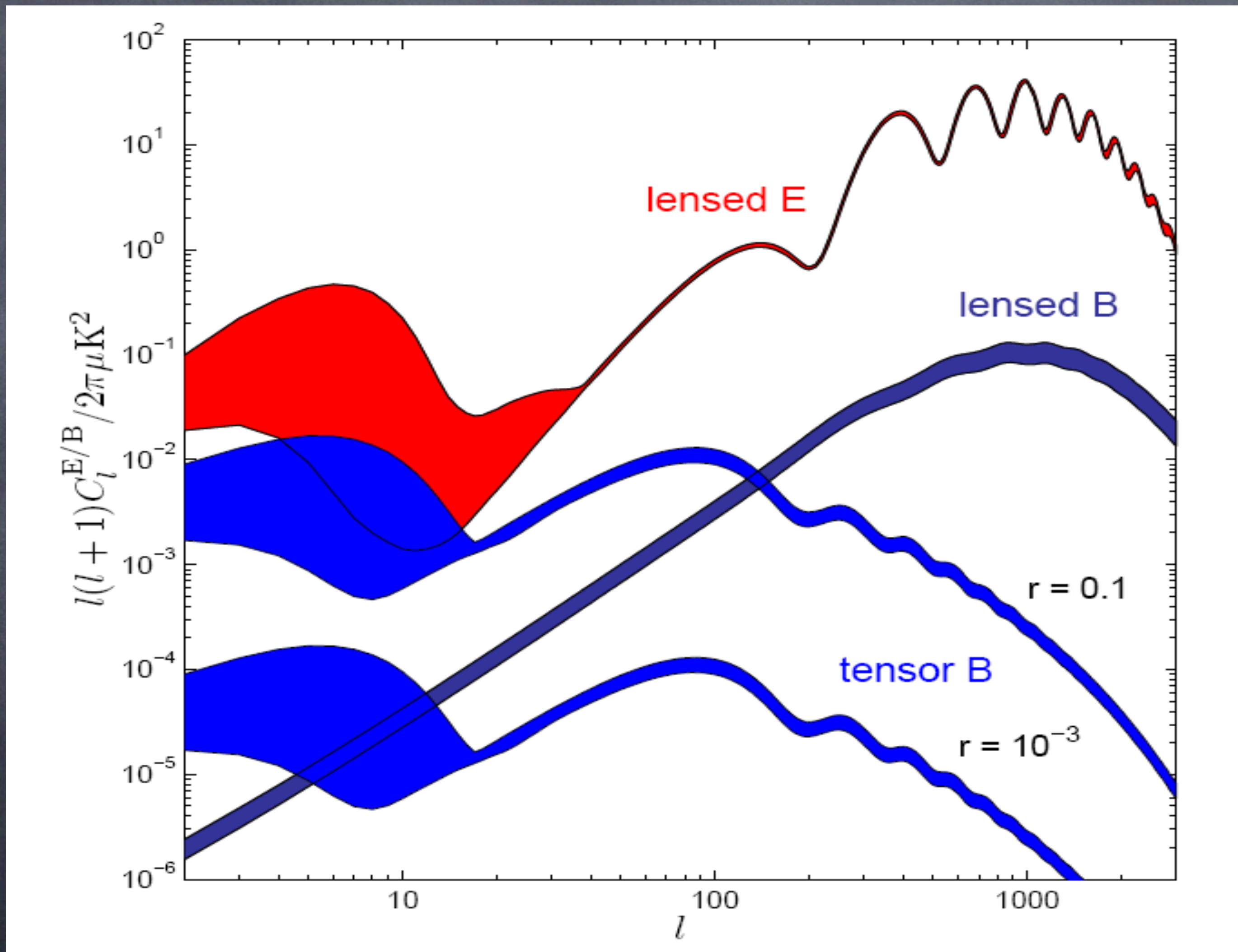
- LENSING POTENTIAL RECONSTRUCTION
- INSTRUMENTAL SYSTEMATICS
- EFFECTS ON LENSING RECONSTRUCTION
- DISTORTIONS OF CMB AND PRIMORDIAL B-MODE DIAGNOSTICS
- SUMMARY

CMB lensing from Large Scale Structure



From: Carlo Baccigalupi

Lensing as foreground to GW B-mode



(Lewis and Challinor 2006)

* Lensing destroys the isotropy of the CMB sky and introduces coupling between CMB harmonic modes otherwise uncorrelated

Deflection angle

$$\tilde{T}(\hat{\mathbf{n}}) = T(\hat{\mathbf{n}} + \mathbf{d}(\hat{\mathbf{n}})),$$

$$[\tilde{Q} \pm i\tilde{U}](\hat{\mathbf{n}}) = [Q \pm iU](\hat{\mathbf{n}} + \mathbf{d}(\hat{\mathbf{n}}))$$

$$d(\hat{\mathbf{n}}) = \nabla\phi(\hat{\mathbf{n}}) \qquad \nabla^2\phi = -2\kappa$$

$$\phi(\hat{\mathbf{n}}) = -2\int_0^{r_0} dr\, \frac{d_A(r_0 - r)}{d_A(r)d_A(r_0)}\,\Phi(r, r\hat{\mathbf{n}})$$

Off-diagonal terms are proportional the lensing potential

$$\langle X^i(\boldsymbol{l}_1) X'^j(\boldsymbol{l}_2)\rangle \equiv (2\pi)^2 \delta_D(\boldsymbol{l}_1 + \boldsymbol{l}_2) C^{ij}_{X^i X'^j}(l_1),$$

$$= f_\alpha(\boldsymbol{l}, \boldsymbol{l}') \phi(\boldsymbol{L})$$

First order

$$f_{XX'}(\boldsymbol{l}_1, \boldsymbol{l}_2) = C^{XX'}_{l_1} {}^1W_{XX'}(\boldsymbol{l}_1, \boldsymbol{l}_2) + C^{XX'}_{l_2} {}^2W_{XX'}(\boldsymbol{l}_1, \boldsymbol{l}_2),$$

Window functions

XX'	${}^1W_{XX'}(\boldsymbol{l}_1, \boldsymbol{l}_2)$	${}^2W_{XX'}(\boldsymbol{l}_1, \boldsymbol{l}_2)$
TT	$(\mathbf{L} \cdot \boldsymbol{l}_1)$	$(\mathbf{L} \cdot \boldsymbol{l}_2)$
TE	$\cos 2(\varphi_{l_1} - \varphi_{l_2})(\mathbf{L} \cdot \boldsymbol{l}_1)$	$(\mathbf{L} \cdot \boldsymbol{l}_2)$
TB	$\sin 2(\varphi_{l_1} - \varphi_{l_2})(\mathbf{L} \cdot \boldsymbol{l}_1)$	0
EE	$\cos 2(\varphi_{l_1} - \varphi_{l_2})(\mathbf{L} \cdot \boldsymbol{l}_1)$	$\cos 2(\varphi_{l_1} - \varphi_{l_2})(\mathbf{L} \cdot \boldsymbol{l}_2)$
EB	$\sin 2(\varphi_{l_1} - \varphi_{l_2})(\mathbf{L} \cdot \boldsymbol{l}_1)$	$\sin 2(\varphi_{l_1} - \varphi_{l_2})(\mathbf{L} \cdot \boldsymbol{l}_2)$
BB	$\cos 2(\varphi_{l_1} - \varphi_{l_2})(\mathbf{L} \cdot \boldsymbol{l}_1)$	$\cos 2(\varphi_{l_1} - \varphi_{l_2})(\mathbf{L} \cdot \boldsymbol{l}_2)$

Quadratic Esimtator: Lensing induced non-Gaussianity

normalization $\langle d_{XX'}(\mathbf{L})\rangle_{\rm CMB} = d(\mathbf{L})$

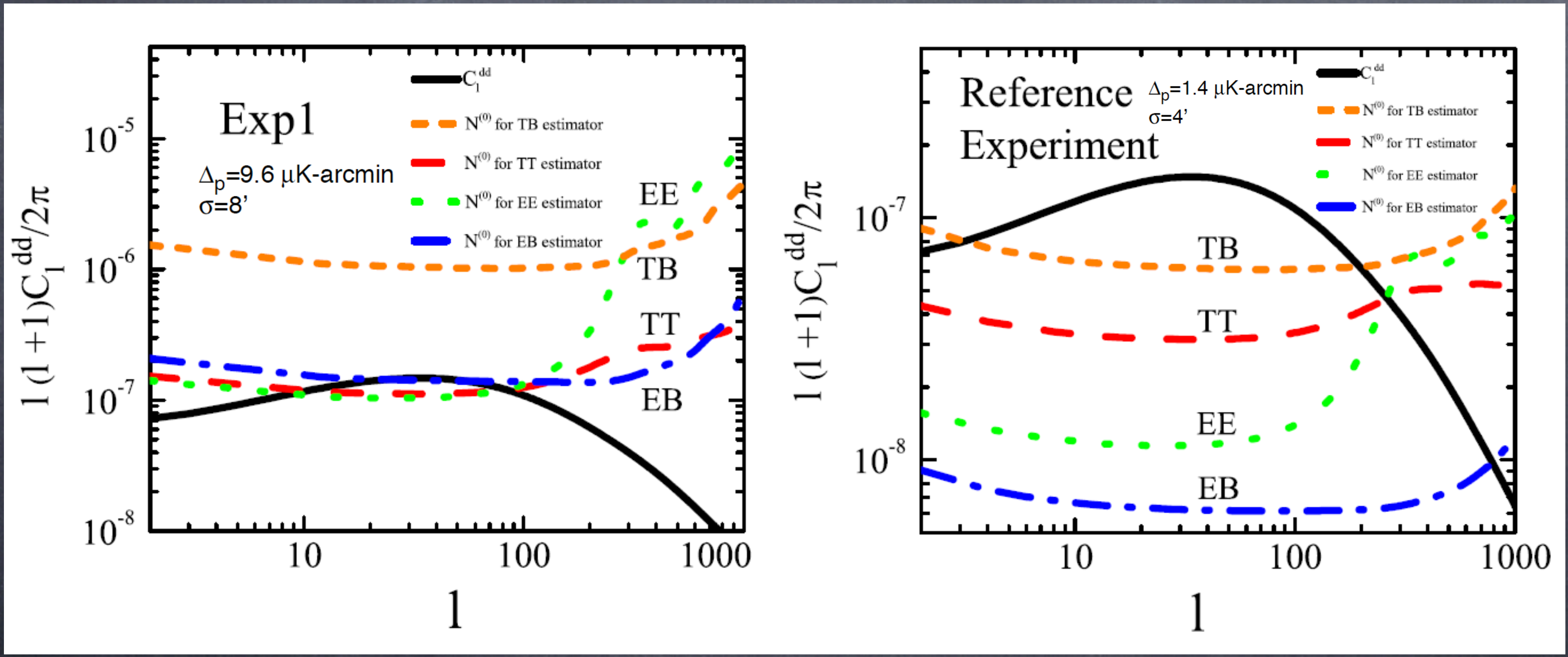
$$d_{XX'}(\mathbf{L}) \equiv \frac{A_{XX'}(L)}{L}\int \frac{d^2\boldsymbol{l}_1}{(2\pi)^2} X^{\rm t}(\boldsymbol{l}_1) X'^{\rm t}(\boldsymbol{l}_2) F_{XX'}(\boldsymbol{l}_1,\boldsymbol{l}_2)$$

Filter function

$$F_{XX'}(\boldsymbol{l}_1,\boldsymbol{l}_2) = \frac{C_{l_1}^{X'X'{\rm t}} C_{l_2}^{XX{\rm t}} f_{XX'}(\boldsymbol{l}_1,\boldsymbol{l}_2) - C_{l_1}^{XX'{\rm t}} C_{l_2}^{XX'{\rm t}} f_{XX'}(\boldsymbol{l}_2,\boldsymbol{l}_1)}{C_{l_1}^{XX{\rm t}} C_{l_2}^{X'X'{\rm t}} C_{l_1}^{X'X'{\rm t}} C_{l_2}^{XX{\rm t}} - (C_{l_1}^{XX'{\rm t}} C_{l_2}^{XX'{\rm t}})^2},$$

Hu and Okamoto (2002)

Dominant Reconstruction Noise



MS, Yadav, Zaldarriaga (2009)

INSTRUMENTAL SYSTEMATIC EFFECTS ON LENSING RECONSTRUCTION

Systematics: challenge for next CMB experiment

* Challenges for CMB lensing detection
  - -astrophysical foregrounds
    (See talks by Osborne, Benoit-Levy / Dechelette)
  - -instrumental systematics (See also Miller's talk)

* Important to estimate and control those spurious signals as well as possible when analyzing upcoming CMB data.

* Instrumental systematics may well be required to reconstruct the lensing potential and/or delense the observed B-mode to push constraints on r

* Two types of systematics:

1, The detector system which distorts the polarization state of the incoming polarized signal
2, Distortion of the CMB signal due to the beam anisotropy

Instrumental Systematics

$$\tilde{T}^{\rm obs}(\hat{\mathbf{n}}) = [1 + a(\hat{\mathbf{n}})]\tilde{T}^{t}(\hat{\mathbf{n}})$$

Calibration systematics

$$\tilde{C}_l^{TT} = \left[1 - \int \frac{d^2\boldsymbol{l}_1}{(2\pi)^2} C_{l_1}^{\phi\phi}(\boldsymbol{l}_1 \cdot \boldsymbol{l})^2\right] C_l^{TT} + \int \frac{d^2\boldsymbol{l}_1}{(2\pi)^2} C_{|\boldsymbol{l}-\boldsymbol{l}_1|}^{TT} C_{l_1}^{\phi\phi}[(\boldsymbol{l} - \boldsymbol{l}_1)\cdot \boldsymbol{l}_1]^2 + \int \frac{d^2\boldsymbol{l}_1}{(2\pi)^2} C_{\boldsymbol{l}-\boldsymbol{l}_1}^{aa} C_{l_1}^{TT}.$$

Bias on power spectrum

$$\langle d_{TT}(\mathbf{L})\rangle_{\rm CMB} = d_{TT}(\mathbf{L}) + \frac{A_{TT}(L)}{L} \times \int \frac{d^2\boldsymbol{l}_1}{(2\pi)^2} F_{TT}(\boldsymbol{l}_1, \boldsymbol{l}_2) a(\mathbf{L})(C_{l_1}^{TT} + C_{l_2}^{TT}),$$

Estimator Bais

MS, Yadav, Zaldarriaga (2009)

Instrumental Systematics of CMB polarization

$$\delta[Q \pm iU](\hat{\mathbf{n}}) = [a \pm i2\omega](\hat{\mathbf{n}})[Q \pm iU](\hat{\mathbf{n}}) + [f_1 \pm i f_2](\hat{\mathbf{n}})[Q \mp iU](\hat{\mathbf{n}}) + [\gamma_1 \pm i\gamma_2](\hat{\mathbf{n}})T(\hat{\mathbf{n}}),$$

Calibration and rotation

Spin flip

Monopole leakage

Differential gain

$$\delta[Q \pm iU](\hat{\mathbf{n}};\sigma) = \sigma\mathbf{p}(\hat{\mathbf{n}})\cdot\nabla[Q \pm iU](\hat{\mathbf{n}};\sigma) + \sigma[d_1 \pm i d_2](\hat{\mathbf{n}})[\partial_1 \pm i\partial_2]T(\hat{\mathbf{n}};\sigma) + \sigma^2 q(\hat{\mathbf{n}})[\partial_1 \pm i\partial_2]^2 T(\hat{\mathbf{n}};\sigma),$$

Pointing

Dipole leakage

(Differential pointing)

Quadruple leakage

Differential ellipticity

Hu, Hedman, Zaldarriaga (2003)

Parameterization of the beam

the beam offset

$$\mathcal{B}(\hat{\mathbf{n}};\mathbf{b},e) = \frac{1}{2\pi\sigma^2(1-e^2)} \exp\left[-\frac{1}{2\sigma^2}\left(\frac{(n_1-b_1)^2}{(1+e)^2} + \frac{(n_2-b_2)^2}{(1-e)^2}\right)\right]$$

Beam ellipticity

Assuming distortions of the beam are relatively small compare to typical scales of the beam on either direction

$$\sigma\mathbf{p} = (\mathbf{b}_a + \mathbf{b}_b)/2\,,$$
$$\sigma\mathbf{d} = (\mathbf{b}_a - \mathbf{b}_b)/2\,,$$
$$e_s = (e_a + e_b)/2\,,$$
$$q = (e_a - e_b)/2\,,$$

Hu, Hedman, Zaldarriaga (2003)

Systematic Induced Bias on deflection angle variance

$$\left\langle\left\langle\langle d_{EB}(\mathbf{L}) \cdot d_{EB}(\mathbf{L}')\rangle_{\rm CMB}\right\rangle_{\rm LSS}\right\rangle_{\rm SYS}$$
$$= \frac{A_{EB}(L)}{L}\frac{A_{EB}(L')}{L'}\int\frac{d^2\boldsymbol{l}_1}{(2\pi)^2}\int\frac{d^2\boldsymbol{l}'_1}{(2\pi)^2}F_{EB}(\boldsymbol{l}_1,\boldsymbol{l}_2)$$
$$\times F_{EB}(\boldsymbol{l}_1{}',\boldsymbol{l}_2{}')\langle\tilde{E}(\boldsymbol{l}_1)^{\rm obs}\tilde{B}(\boldsymbol{l}_2)^{\rm obs}\tilde{E}(\boldsymbol{l}'_1)^{\rm obs}\tilde{B}(\boldsymbol{l}'_2)^{\rm obs}\rangle$$
$$= (2\pi)^2\delta_D(\mathbf{L}+\mathbf{L}')[C^{dd}(L) + N^{(0)}_{EB,EB}(L) + N^{(1)}_{EB,EB}(L)$$
$$+ N^{(S)}_{EB,EB}(L) + \ldots],$$

Systematic induced variance

MS, Yadav, Zaldarriag (2009)

$$N^{(S)}_{EB,EB}(L) = \frac{A_{EB}(L)}{L}\frac{A_{EB}(L')}{L'}\int\frac{d^2\boldsymbol{l}_1}{(2\pi)^2}\int\frac{d^2\boldsymbol{l}'_1}{(2\pi)^2}F_{EB}(\boldsymbol{l}_1,\boldsymbol{l}_2)F_{EB}(\boldsymbol{l}_1{}',\boldsymbol{l}_2{}')\Big\{C^{EE}_{l_1}C^{EE}_{l'_1}\Big[\sum_S^{P\text{-distortion}} \boxed{C^{SS}_{\boldsymbol{l}_1+\boldsymbol{l}_2}} W^S_B(\boldsymbol{l}_2,-\boldsymbol{l}_1)W^S_B(\boldsymbol{l}'_2,-\boldsymbol{l}'_1)$$

$$+\sum_S^{P\text{-distortion}} \boxed{C^{SS}_{\boldsymbol{l}_1+\boldsymbol{l}'_2}} W^S_B(\boldsymbol{l}_2,-\boldsymbol{l}'_1)W^S_B(\boldsymbol{l}'_2,-\boldsymbol{l}_1)\Big] + C^{TE}_{l_1}C^{TE}_{l'_1}\Big[\sum_S^{T\text{-leakage}} \boxed{C^{SS}_{\boldsymbol{l}_1+\boldsymbol{l}_2}} W^S_B(\boldsymbol{l}_2,-\boldsymbol{l}_1)W^S_B(\boldsymbol{l}'_2,-\boldsymbol{l}'_1)$$

$$+\sum_S^{T\text{-leakage}} \boxed{C^{SS}_{\boldsymbol{l}_1+\boldsymbol{l}'_2}} W^S_B(\boldsymbol{l}_2,-\boldsymbol{l}'_1)W^S_B(\boldsymbol{l}'_2,-\boldsymbol{l}_1)\Big]\Big\}.$$

Type of S	$W^S_B(\boldsymbol{l}_1,\boldsymbol{l}_2)$	$W^S_E(\boldsymbol{l}_1,\boldsymbol{l}_2)$
Calibration a	$\sin[2(\varphi_{l_2}-\phi_L)]$	$\cos[2(\varphi_{l_2}-\phi_L)]$
Rotation ω	$2\cos[2(\varphi_{l_2}-\varphi_L)]$	$-2\sin[2(\varphi_{l_2}-\varphi_L)]$
Pointing p_a	$\sigma(\boldsymbol{l}_2\times\hat{\boldsymbol{l}}_1)\cdot\hat{\mathbf{z}}\sin[2(\varphi_{l_2}-\varphi_L)]$	$\sigma(\boldsymbol{l}_2\cdot\hat{\boldsymbol{l}}_1)\sin[2(\varphi_{l_2}-\varphi_l)]$
Pointing p_b	$\sigma(\boldsymbol{l}_2\cdot\hat{\boldsymbol{l}}_1)\sin[2(\varphi_{l_2}-\varphi_l)]$	$-\sigma(\boldsymbol{l}_2\times\hat{\boldsymbol{l}}_1)\cdot\hat{\mathbf{z}}\sin[2(\varphi_{l_2}-\varphi_L)]$
Flip f_a	$\sin[2(2\varphi_{l_1}-\varphi_{l_2}-\varphi_L)]$	$\cos[2(2\varphi_{l_1}-\varphi_{l_2}-\varphi_L)]$
Flip f_b	$\cos[2(2\varphi_{l_1}-\varphi_{l_2}-\varphi_L)]$	$-\sin[2(2\varphi_{l_1}-\varphi_{l_2}-\varphi_L)]$
Monopole γ_a	$\sin[2(\varphi_{l_1}-\varphi_l)]$	$\cos[2(\varphi_{l_1}-\varphi_l)]$
Monopole γ_b	$\cos[2(\varphi_{l_1}-\varphi_l)]$	$-\sin[2(\varphi_{l_1}-\varphi_l)]$
Dipole d_a	$-(l_2\sigma)\cos[\varphi_{l_1}+\varphi_{l_2}-2\varphi_l]$	$(l_2\sigma)\sin[\varphi_{l_1}+\varphi_{l_2}-2\varphi_l]$
Dipole d_b	$(l_2\sigma)\sin[\varphi_{l_1}+\varphi_{l_2}-2\varphi_l]$	$(l_2\sigma)\cos[\varphi_{l_1}+\varphi_{l_2}-2\varphi_l]$
Quadrupole q	$-(l_2\sigma)^2\sin[2(\varphi_{l_2}-\varphi_l)]$	$-(l_2\sigma)^2\cos[2(\varphi_{l_2}-\varphi_l)]$

Model the instrumental systematics field

White noise above coherence length

$$C_l^{SS} = C_0 \exp(-l(l+1)\alpha_S^2)$$

Coherence length

$$C_0 = A_S^2 \left[\int \frac{d^2 l}{(2\pi)^2} \exp(-l(l+1)\alpha_S^2) \right]^{-1}$$

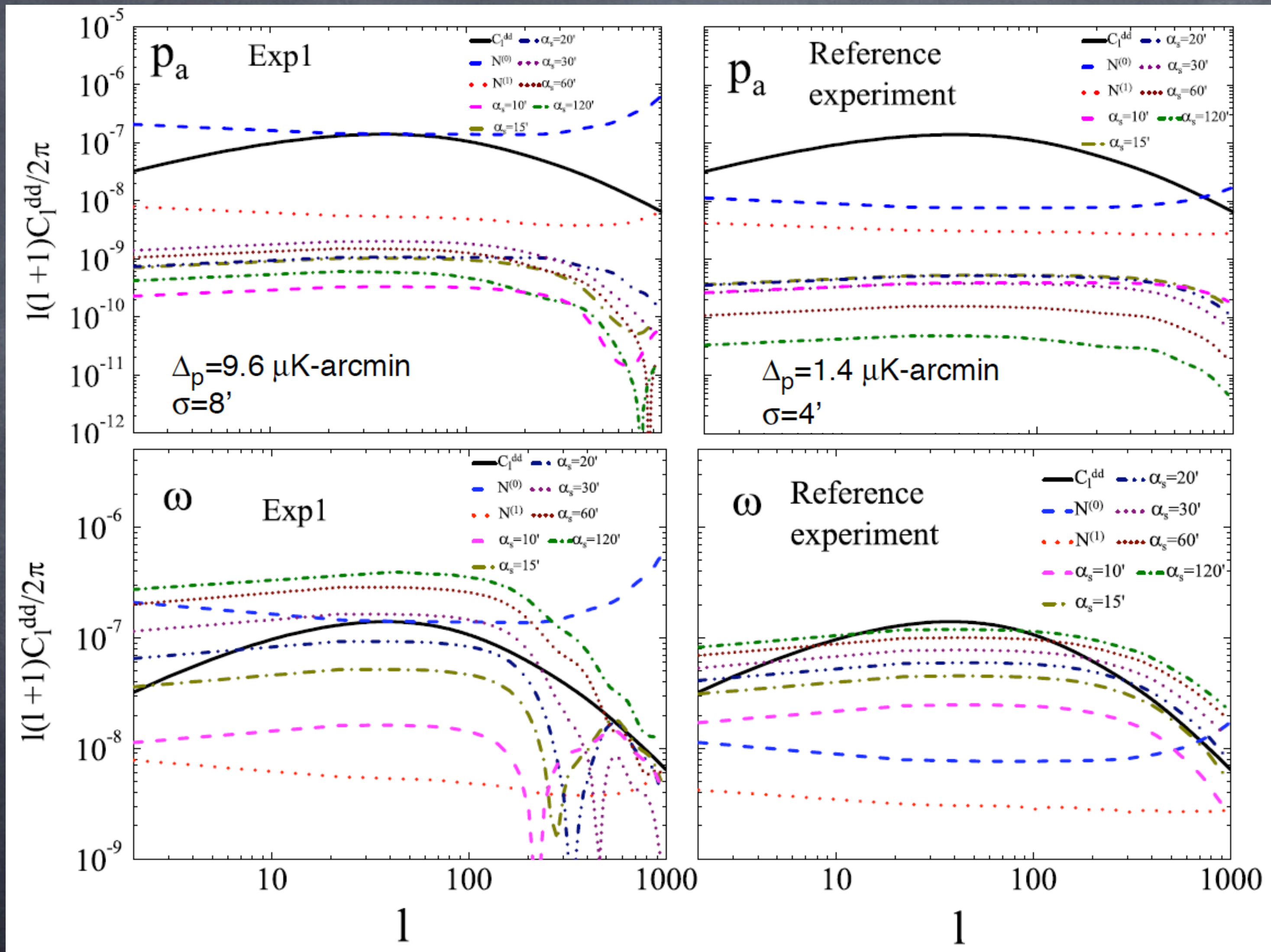
Instrumental Systematics Fluctuation amplitude

$$\alpha_S(\mathbf{l}) = \tilde{\alpha}_S\{1 + B\cos^n(\phi_l + \psi)\}, \qquad \text{Model 1}$$

$$\alpha_S(\mathbf{l}) = \tilde{\alpha}_S\{1 + B\cos(n\phi_l + \psi)\}, \qquad \text{Model 2}$$

Hu, Hedman, Zaldarriaga (2003)
MS, Yadav, Zaldarriag (2009)

Pointing and rotation as an examples



MS, Yadav, Zaldarriaga (2009)

Systematic contamination for lensing and B-mode detection

Compare systematics requirement for lensing and B-mode

Coherence length of systematics field

Δ_p=9.6 μK-arcmin, σ=8' v.s. Δ_p=1.4 μK-arcmin, σ=4'

Type	Experiment 1						Reference					
	Lensing		B modes				Lensing		B modes			
	$\ell = 40$		3.0×10^{16} GeV		1.0×10^{16} GeV		$\ell = 40$		3.0×10^{16} GeV		1.0×10^{16} GeV	
	$\alpha_s = 10'$	$\alpha_s = 120'$	$\alpha_s = 10'$	$\alpha_s = 120'$	$\alpha_s = 10'$	$\alpha_s = 120'$	$\alpha_s = 10'$	$\alpha_s = 120'$	$\alpha_s = 10'$	$\alpha_s = 120'$	$\alpha_s = 10'$	$\alpha_s = 120'$
Calibration a	1.04	0.55	0.549	0.468	0.061	0.052	0.86	1.56	0.486	0.468	0.054	0.052
Rotation w	0.30	0.061	0.27	0.207	0.030	0.023	0.24	0.11	0.243	0.198	0.027	0.022
Pointing p_a	2.10	1.57	8.55	6.12	0.95	0.68	1.93	5.55	15.3	12.24	1.70	1.36
Pointing p_b	1.39	2.76	1.08	6.57	0.12	0.73	2.31	9.74	1.71	13.14	0.19	1.46
Flip f_a	1.08	0.17	0.549	0.441	0.061	0.049	0.62	0.31	0.486	0.432	0.054	0.048
Flip f_b	0.61	0.17	0.531	0.360	0.059	0.040	0.58	0.31	0.477	0.36	0.053	0.040
Monopole γ_a	0.114	0.024	0.021	0.0058	0.0023	0.00064	0.13	0.013	0.0207	0.0058	0.0023	0.00064
Monopole γ_b	0.114	0.036	0.014	0.0034	0.0016	0.00038	0.64	0.12	0.0144	0.0034	0.0016	0.00038
Dipole d_a	0.82	0.11	0.085	0.060	0.0094	0.0067	0.97	0.39	0.153	0.117	0.017	0.013
Dipole d_b	0.55	0.092	0.85	0.063	0.0094	0.0070	1.02	0.33	0.153	0.126	0.017	0.014
Quadrupole q	1.58	0.78	0.162	0.558	0.018	0.062	2.38	5.47	0.495	2.25	0.055	0.25

MS, A. Yadav, M, Zaldarriaga, PRD, 2009

Distortions of the Primary CMB field along the line of sight

1. Modulation	$\delta(Q \pm iU)(n) = a(n)(Q \pm iU)(n)$	Patchy reionization, Hot and cold spots	Gain
2. Rotation	$\delta(Q + iU)(n) = 2i\omega(n)(Q + iU)(n)$	Magnetic fields Parity violation Fine structure cost.	Pixel rotation
3. Spin-flip	$\delta[Q + iU](n) = [f_1 + if_2](n)[Q - iU](n)$		Spin Flip
4. Monopole leakage	$\delta[Q + iU](n) = [\gamma_1 + i\gamma_2](n)T(n)$		Differential gain
5. Dipole leakage	$\delta[Q + iU](n) = \sigma[d_1 + id_2](n)[\partial_1 + \partial_2]T(n)$		Different. pointing
6. Quadrupole leakage	$\delta[Q + iU](n) = \sigma^2 q(n)[\partial_1 + \partial_2]^2 T(n)$		Different. ellipticity
7. Deflection	$\delta[Q + iU](n) = \sigma p(n) \cdot \nabla T(n)$	Lensing (gradient- and curl- type) Cosmic string	Pointing

(see Namikawa's talk)

Hu, Hedman, Zaldarriaga (2003)

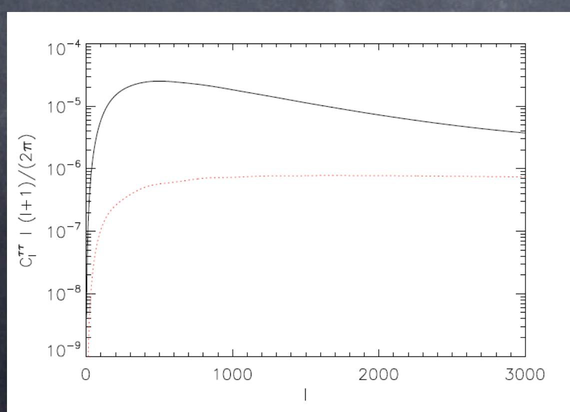
Yadav, Su, Zaldarriaga (2010)

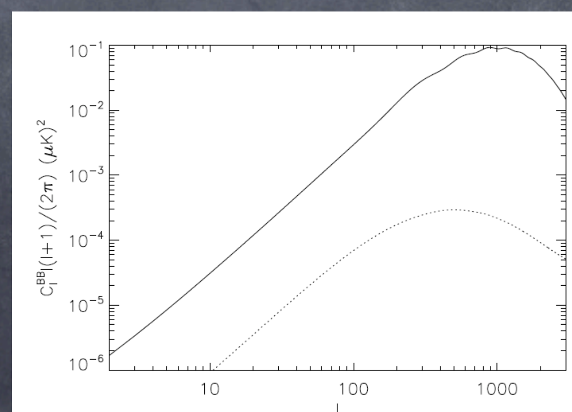
Patchy Reionization

* Three main sources:
- (a) kSZ from peculiar motion of ionized regions
- (b) ionized bubbles scatter the local CMB temperature quadrupole
- (c) LOS dependent Thomson scattering optical depth

$$T(\hat{\mathbf{n}}) = e^{-\delta\tau(\hat{\mathbf{n}})}\tilde{T}(\hat{\mathbf{n}}),$$
$$(Q \pm iU)(\hat{\mathbf{n}}) = e^{-\delta\tau(\hat{\mathbf{n}})}(\tilde{Q} \pm i\tilde{U})(\hat{\mathbf{n}})$$

$$\tau(\hat{\mathbf{n}}) = c\int \frac{a\,dz}{H(z)}\sigma_T\,\bar{n}_e(z)\,\left[1 + \delta_b(\hat{\mathbf{n}}, z) + \delta_x(\hat{\mathbf{n}}, z)\right]$$





Dvorkin and Smith (2009)

Spatially Dependent Rotation

* The plane of linear polarization of CMB fields can be rotated due to interactions which introduce a different dispersion relation for the left and right circularly polarized modes (non-zero TB and EB power spectrum)

* Two main sources:

- (a) Faraday rotation due to interaction with background magnetic fields, (primordial origin of magnetic field)
- (b) Interactions with pseudoscalar fields

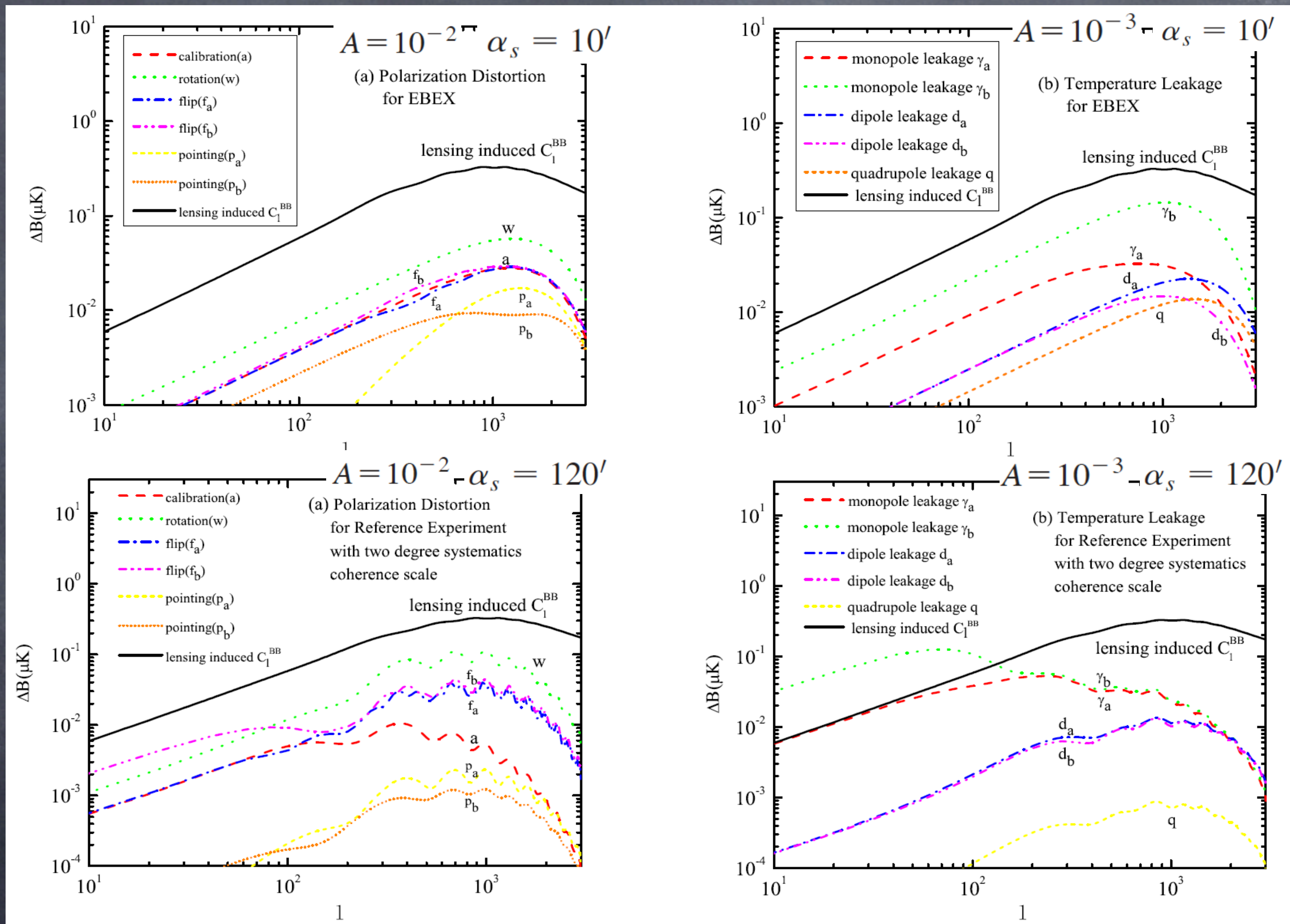
* Another motivation: Control instrumental systematics on primordial B-mode detection, lensing/rotation reconstruction etc.

Example of interaction: $\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 - V(\phi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{\phi}{4f}F_{\mu\nu}\tilde{F}^{\mu\nu}$

Induced rotation angle: $\alpha = \frac{1}{M}\int d\tau\dot{\phi}$

Kamionkowski (2008);
Yadav, Biswas, Su & Zaldarriaga (2009)

Distortion induced B-mode power spectrum



(See Hu et al. 2003)

Spatially dependent Distortion of CMB

Question: If we detect B-modes, how would we know the cause of B-modes? What level of distortions change the observed CMB maps?

- We can construct 11 estimators (one for each distortion), which using the observed E and B fields reconstructs all the 11 Distortion fields.

$$\langle T(\mathbf{l_1})B(\mathbf{l_2})\rangle = f^{\mathcal{D}}_{TB}(\mathbf{l_1},\mathbf{l_2})\ \mathcal{D}(\mathbf{l_1}-\mathbf{l_2})$$

$$\langle E(\mathbf{l_1})B(\mathbf{l_2})\rangle = f^{\mathcal{D}}(\mathbf{l_1},\mathbf{l_2})\ \mathcal{D}(\mathbf{l_1}-\mathbf{l_2})$$

$$C_l^{BB} = \int d^2l'\, C^{\mathcal{D}\mathcal{D}}_{l-l'}\, W^{\mathcal{D}}(l,l')$$

- Non-zero off-diagonal <EB>
- New B-modes power spectrum
  - contamination to primordial B-modes
- <EB> ∝ distortion

  <BB> ∝ $(\text{distortion})^2$

Kamionkowski (2008);
Yadav, Biswas, Su & Zaldarriaga (2009)
Gluscevic, Kamionkowski, Cooray (2009)
Yadav, Su, & Zaldarriaga (2010)

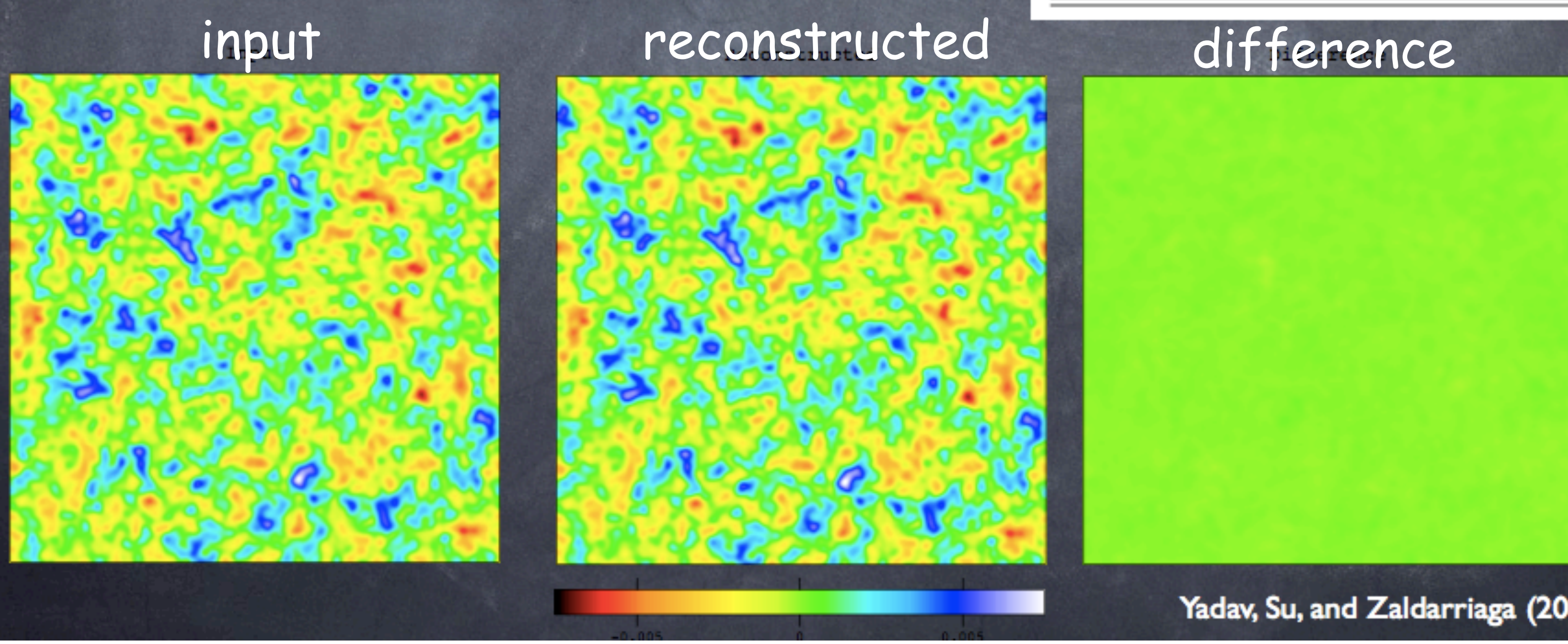
Minimum Variance Estimator for Distortion Fields

$$\langle E(\mathbf{l}_1)B(\mathbf{l}_2)\rangle = f^{\mathcal{D}}(\mathbf{l}_1,\mathbf{l}_2)\ \mathcal{D}(\mathbf{l}_1-\mathbf{l}_2)$$

$$\hat{\mathcal{D}}(\mathbf{L}) = \frac{1}{N(L)}\int \frac{d^2\mathbf{l}_1}{(2\pi)^2} E(\mathbf{l}_1)B(\mathbf{l}_2)\frac{f^{\mathcal{D}}(\mathbf{l}_1,\mathbf{l}_2)}{C^{EE}_{l_1}C^{BB}_{l_2}}$$

Normalization to make the estimator unbiased

Distortion $\mathcal{D}$	$f^{\mathcal{D}}_{EB}(\mathbf{l}_1,\mathbf{l}_2)$
Modulation a	$\tilde{C}^{EE}_{l_1}\sin 2(\varphi_{\mathbf{l}_1}-\varphi_{\mathbf{l}_2})$
Rotation ω	$2\tilde{C}^{EE}_{l_1}\cos 2(\varphi_{\mathbf{l}_1}-\varphi_{\mathbf{l}_2})$
Monopole leakage γ_1	$\tilde{C}^{TE}_{\ell_1}\sin 2(\varphi_{\mathbf{L}}-\varphi_{\mathbf{l}_2})$
Monopole leakage γ_2	$\tilde{C}^{TE}_{\ell_1}\cos 2(\varphi_{\mathbf{L}}-\varphi_{\mathbf{l}_2})$
Spin-flip f_1	$\tilde{C}^{EE}_{\ell_1}\sin 2(2\varphi_{\mathbf{L}}-\varphi_{\mathbf{l}_1}-\varphi_{\mathbf{l}_2})$
Spin-flip f_2	$\tilde{C}^{EE}_{\ell_1}\cos 2(2\varphi_{\mathbf{L}}-\varphi_{\mathbf{l}_1}-\varphi_{\mathbf{l}_2})$
Dipole leakage d_1	$\tilde{C}^{TE}_{\ell_1}(\ell_1\sigma)\cos(\varphi_{\mathbf{L}}+\varphi_{\mathbf{l}_1}-2\varphi_{\mathbf{l}_2})$
Dipole leakage d_2	$-\tilde{C}^{TE}_{\ell_1}(\ell_1\sigma)\sin(\varphi_{\mathbf{L}}+\varphi_{\mathbf{l}_1}-2\varphi_{\mathbf{l}_2})$
Quadrupole leakage q	$-\tilde{C}^{TE}_{\ell_1}(\ell_1\sigma)^2\sin 2(\varphi_{\mathbf{l}_1}-\varphi_{\mathbf{l}_2})$
Pointing p_1	$-\tilde{C}^{EE}_{\ell_1}\sigma(\ell_1\times\hat{\mathbf{L}})\sin 2(\varphi_{\mathbf{l}_1}-\varphi_{\mathbf{l}_2})$
Pointing p_2	$-\tilde{C}^{EE}_{\ell_1}\sigma(\mathbf{l}_1\cdot\hat{\mathbf{L}})\sin 2(\varphi_{\mathbf{l}_1}-\varphi_{\mathbf{l}_2})$
Lensing ϕ	$\tilde{C}^{EE}_{\ell_1}(\mathbf{L}\cdot\mathbf{l}_1)\sin 2\varphi_{\mathbf{l}_1\mathbf{l}_2}$



Yadav, Su, and Zaldarriaga (2010)

Estimator noise for distortion fields

$$\langle \hat{\mathcal{D}}_{XX'}(\mathbf{L})\hat{\mathcal{D}}_{YY'}(\mathbf{L}')\rangle_{\text{CMB, distortion}} = (2\pi)^2\delta(\mathbf{L}+\mathbf{L}')\{C_L^{\mathcal{D}\mathcal{D}} + N^{\mathcal{D}}_{XX',YY'}(L) + N^{(1)\mathcal{D},}_{XX',YY'}(L) + N^{(2),\mathcal{D}}_{XX',YY'}(L) + \cdots\},$$

$$N^{\mathcal{D}}_{XX',YY'}(L) = (2\pi)^2 A^{\mathcal{D}}_{XX'}(L) A^{\mathcal{D}}_{YY'}(L) \int \frac{d^2\mathbf{l}_1}{(2\pi)^2} F^{\mathcal{D}}_{XX'}(\mathbf{l}_1,\mathbf{l}_2) \times [F^{\mathcal{D}}_{YY'}(\mathbf{l}_1,\mathbf{l}_2) C^{XY}_{\ell_1} C^{X'Y'}_{\ell_2} + F^{\mathcal{D}}_{YY'}(\mathbf{l}_1,\mathbf{l}_2) C^{XY'}_{\ell_1} C^{X'Y}_{\ell_2}],$$

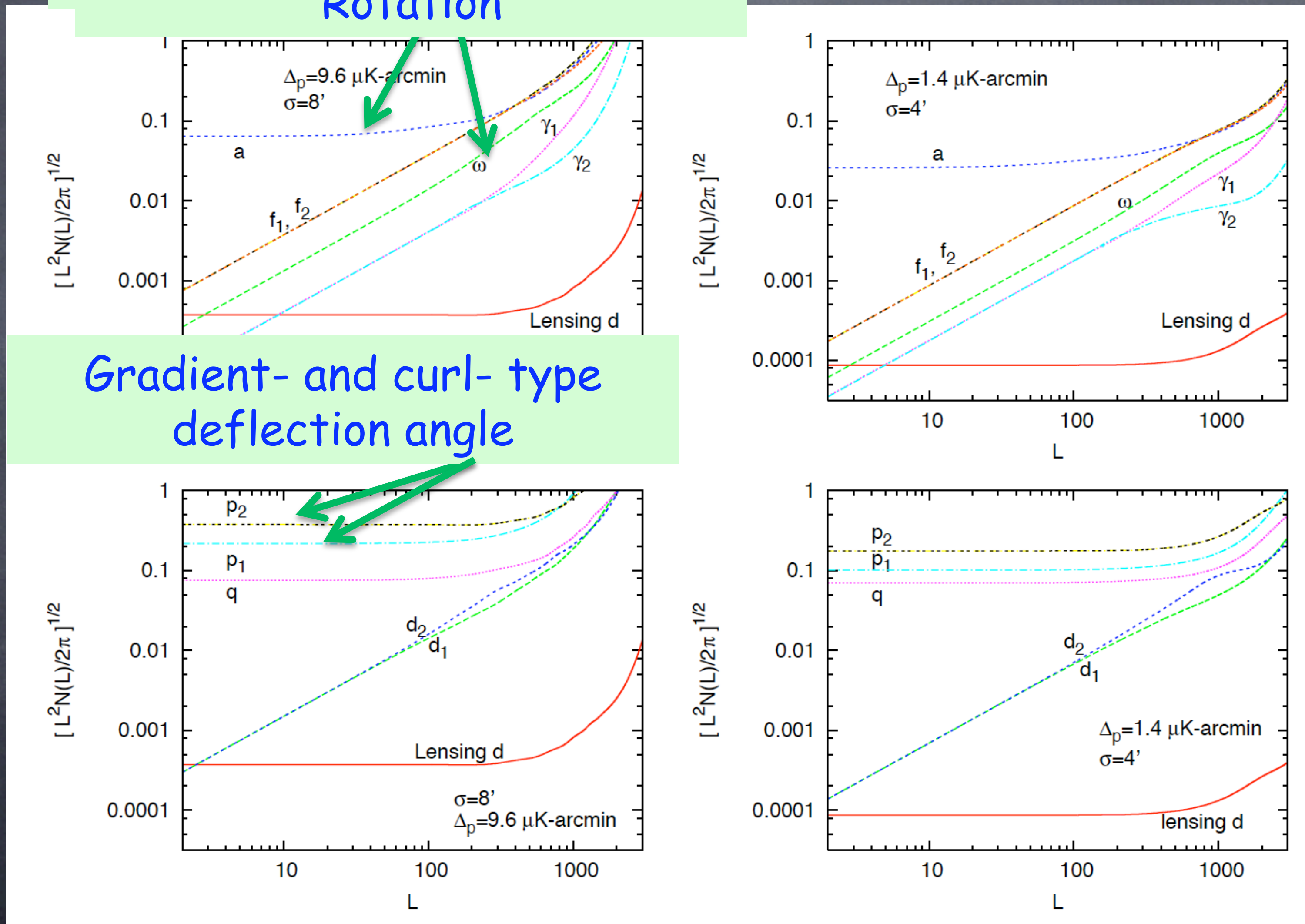
$$F^{\mathcal{D}}_{XX'}(\mathbf{l}_1,\mathbf{l}_2) = \frac{f^{\mathcal{D}}_{XX'}(\mathbf{l}_1,\mathbf{l}_2)}{C^{XX}_{\ell_1} C^{X'X'}_{\ell_2}}$$

TABLE I. Filters, $f^{\mathcal{D}}_{XX'}(\mathbf{l}_1,\mathbf{l}_2)$.

$\mathcal{D}$	$f^{\mathcal{D}}_{EB}(\mathbf{l}_1,\mathbf{l}_2)$	$f^{\mathcal{D}}_{TB}(\mathbf{l}_1,\mathbf{l}_2)$	$W^B_{\mathcal{D}}(\mathbf{l}_1,\mathbf{l}_2)$	$W^E_{\mathcal{D}}(\mathbf{l}_1,\mathbf{l}_2)$
a	$\tilde{C}^{EE}_{l_1}\sin 2(\varphi_{\mathbf{l}_1}-\varphi_{\mathbf{l}_2})$	$\tilde{C}^{TE}_{l_1}\sin 2(\varphi_{\mathbf{l}_1}-\varphi_{\mathbf{l}_2})$	$\sin[2(\varphi_{\mathbf{l}_2}-\varphi_{\mathbf{L}})]$	$\cos[2(\varphi_{\mathbf{l}_2}-\varphi_{\mathbf{L}})]$
ω	$2\tilde{C}^{EE}_{l_1}\cos 2(\varphi_{\mathbf{l}_1}-\varphi_{\mathbf{l}_2})$	$2\tilde{C}^{TE}_{l_1}\cos 2(\varphi_{\mathbf{l}_1}-\varphi_{\mathbf{l}_2})$	$2\cos[2(\varphi_{\mathbf{l}_2}-\varphi_{\mathbf{L}})]$	$-2\sin[2(\varphi_{\mathbf{l}_2}-\varphi_{\mathbf{L}})]$
γ_1	$\tilde{C}^{TE}_{l_1}\sin 2(\varphi_{\mathbf{L}}-\varphi_{\mathbf{l}_2})$	$\tilde{C}^{TT}_{l_1}\sin 2(\varphi_{\mathbf{L}}-\varphi_{\mathbf{l}_2})$	$\sin[2(\varphi_{\mathbf{l}_1}-\varphi_{\mathbf{L}})]$,	$\cos[2(\varphi_{\mathbf{l}_1}-\varphi_{\mathbf{L}})]$
γ_2	$\tilde{C}^{TE}_{l_1}\cos 2(\varphi_{\mathbf{L}}-\varphi_{\mathbf{l}_2})$	$\tilde{C}^{TT}_{l_1}\cos 2(\varphi_{\mathbf{L}}-\varphi_{\mathbf{l}_2})$	$\cos[2(\varphi_{\mathbf{l}_1}-\varphi_{\mathbf{L}})]$,	$-\sin[2(\varphi_{\mathbf{l}_1}-\varphi_{\mathbf{L}})]$
f_1	$\tilde{C}^{EE}_{l_1}\sin 2(2\varphi_{\mathbf{L}}-\varphi_{\mathbf{l}_1}-\varphi_{\mathbf{l}_2})$	$\tilde{C}^{TE}_{l_1}\sin 2(2\varphi_{\mathbf{L}}-\varphi_{\mathbf{l}_1}-\varphi_{\mathbf{l}_2})$	$\sin[2(2\varphi_{\mathbf{l}_1}-\varphi_{\mathbf{l}_2}-\varphi_{\mathbf{L}})]$	$\cos[2(2\varphi_{\mathbf{l}_1}-\varphi_{\mathbf{l}_2}-\varphi_{\mathbf{L}})]$
f_2	$\tilde{C}^{EE}_{l_1}\cos 2(2\varphi_{\mathbf{L}}-\varphi_{\mathbf{l}_1}-\varphi_{\mathbf{l}_2})$	$\tilde{C}^{TE}_{l_1}\cos 2(2\varphi_{\mathbf{L}}-\varphi_{\mathbf{l}_1}-\varphi_{\mathbf{l}_2})$	$\cos 2(2\varphi_{\mathbf{l}_1}-\varphi_{\mathbf{l}_2}-\varphi_{\mathbf{L}})$	$-\sin 2(2\varphi_{\mathbf{l}_1}-\varphi_{\mathbf{l}_2}-\varphi_{\mathbf{L}})$
d_1	$\tilde{C}^{TE}_{l_1}(l_1\sigma)\cos(\varphi_{\mathbf{L}}+\varphi_{\mathbf{l}_1}-2\varphi_{\mathbf{l}_2})$	$\tilde{C}^{TT}_{l_1}(l_1\sigma)\cos(\varphi_{\mathbf{L}}+\varphi_{\mathbf{l}_1}-2\varphi_{\mathbf{l}_2})$	$-(\mathbf{l}_2\sigma)\cos[\varphi_{\mathbf{l}_1}+\varphi_{\mathbf{l}_2}-2\varphi_l]$	$-(\mathbf{l}_2\sigma)\sin[\varphi_{\mathbf{l}_1}+\varphi_{\mathbf{l}_2}-2\varphi_L]$
d_2	$-\tilde{C}^{TE}_{l_1}(l_1\sigma)\sin(\varphi_{\mathbf{L}}+\varphi_{\mathbf{l}_1}-2\varphi_{\mathbf{l}_2})$	$-\tilde{C}^{TT}_{l_1}(l_1\sigma)\sin(\varphi_{\mathbf{L}}+\varphi_{\mathbf{l}_1}-2\varphi_{\mathbf{l}_2})$	$(l_2\sigma)\sin[\varphi_{\mathbf{l}_1}+\varphi_{\mathbf{l}_2}-2\varphi_{\mathbf{L}}]$	$(l_2\sigma)\cos[\varphi_{\mathbf{l}_1}+\varphi_{\mathbf{l}_2}-2\varphi_{\mathbf{L}}]$
q	$-\tilde{C}^{TE}_{l_1}(l_1\sigma)^2\sin 2(\varphi_{\mathbf{l}_1}-\varphi_{\mathbf{l}_2})$	$-\tilde{C}^{TT}_{l_1}(l_1\sigma)^2\sin 2(\varphi_{\mathbf{l}_1}-\varphi_{\mathbf{l}_2})$	$-(l_2\sigma)^2\sin[2(\varphi_{\mathbf{l}_2}-\varphi_{\mathbf{L}})]$	$-(l_2\sigma)^2\cos[2(\varphi_{\mathbf{l}_2}-\varphi_{\mathbf{L}})]$
p_1	$-\tilde{C}^{EE}_{l_1}\sigma(\mathbf{l}_1\times\hat{\mathbf{L}})\sin 2(\varphi_{\mathbf{l}_1}-\varphi_{\mathbf{l}_2})$	$-\tilde{C}^{TT}_{l_1}\sigma(\mathbf{l}_1\times\hat{\mathbf{L}})\sin 2(\varphi_{\mathbf{l}_1}-\varphi_{\mathbf{l}_2})1$	$\sigma(\mathbf{l}_2\times\hat{\mathbf{l}}_1)\cdot\hat{\mathbf{z}}\sin[2(\varphi_{\mathbf{l}_2}-\varphi_{\mathbf{L}})]$	$\sigma(\mathbf{l}_2\cdot\hat{\mathbf{l}}_1)\sin[2(\varphi_{\mathbf{l}_2}-\varphi_{\mathbf{L}})]$
p_2	$-\tilde{C}^{EE}_{l_1}\sigma(\mathbf{l}_1\cdot\hat{\mathbf{L}})\sin 2(\varphi_{\mathbf{l}_1}-\varphi_{\mathbf{l}_2})$	$-\tilde{C}^{TT}_{l_1}\sigma(\mathbf{l}_1\cdot\hat{\mathbf{L}})\sin 2(\varphi_{\mathbf{l}_1}-\varphi_{\mathbf{l}_2})$	$\sigma(\mathbf{l}_2\cdot\hat{\mathbf{l}}_1)\sin[2(\varphi_{\mathbf{l}_2}-\varphi_{\mathbf{L}})]$	$\sigma(\mathbf{l}_2\times\hat{\mathbf{l}}_1)\cdot\hat{\mathbf{z}}\sin[2(\varphi_{\mathbf{l}_2}-\varphi_{\mathbf{L}})]$

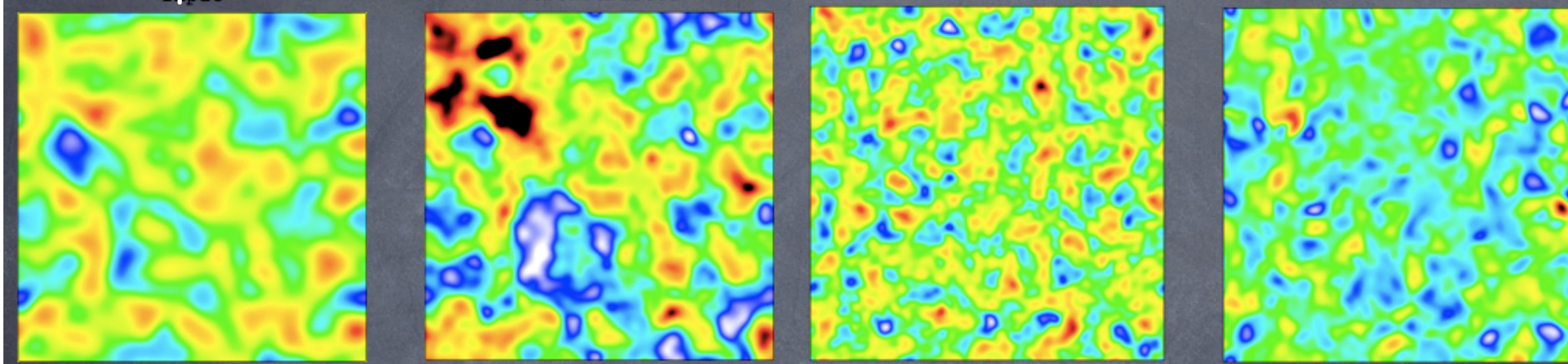
Rotation

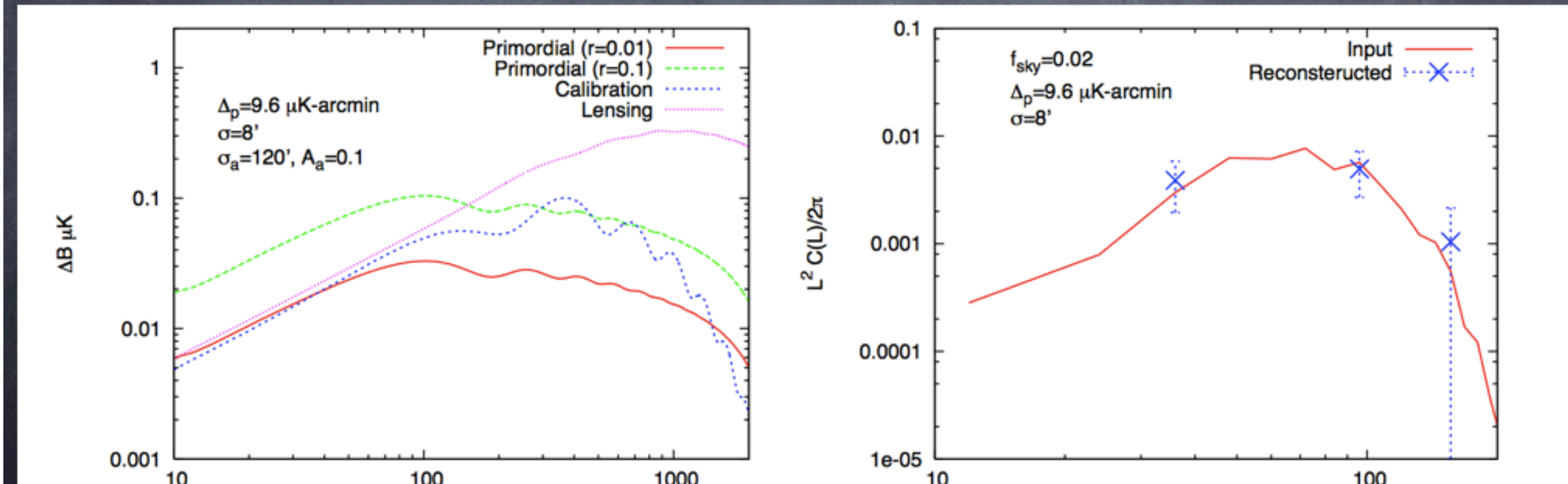
Gradient- and curl- type deflection angle



Yadav, Su, and Zaldarriaga (2010)

Estimator in Action





Yadav, Su, and Zaldarriaga (2010)

The distortions will be detected by minimum variance estimators before the distortions show up in B-modes power spectrum

Distortion type

Compare with minimum detected rms with distortion estimators

Maximum allowed rms of distortion fluctuations

Column 1	Column 2				Column 3	
	$\left(\frac{A_D^{max}}{A_D^{min}}\right)\left(\frac{f_{sky}}{f_\star}\right)^{1/4}\left(\frac{r}{r_\star}\right)^{1/2}$				Maximum allowed *rms* A_D^{max} for B-modes detection [17] $\left(\frac{r}{0.005}\right)^{1/2}$	
Distortion Type	EXP-balloon ($f_\star = 0.01, r_\star = 0.05$)		CMBPol ($f_\star = 0.8, r_\star = 0.005$)			
	$\sigma_s = 10'$	$\sigma_s = 120'$	$\sigma_s = 10'$	$\sigma_s = 120'$	$\sigma_s = 10'$	$\sigma_s = 120'$
Rotation ω	3.4	11.9	16.72	49.02	0.015	0.011
Modulation a	6.0	5.13	25.46	12.73	0.06	0.049
Monopole leakage γ_1	1.9	2.13	4.75	4.65	0.0023	0.0006
Dipole leakage γ_2	2.0	1.7	6.46	3.9	0.0019	0.0005
Spin flip f_1	6.2	17.9	29.35	73.15	0.061	0.046
Spin flip f_2	6.3	17.6	28.7	71.53	0.059	0.045
Dipole leakage d_1	2.2	5.23	5.4	10.54	0.0077	0.0053
Dipole leakage d_2	1.7	5.38	3.8	11.11	0.0077	0.0056
Quadrupole q	1.8	4.1	3.32	3.55	0.0124	0.0394
Deflection p_1	38.2	19.4	132.7	40.3	0.75	0.53
Deflection p_2	4.4	15.5	10.8	24.8	0.098	0.57

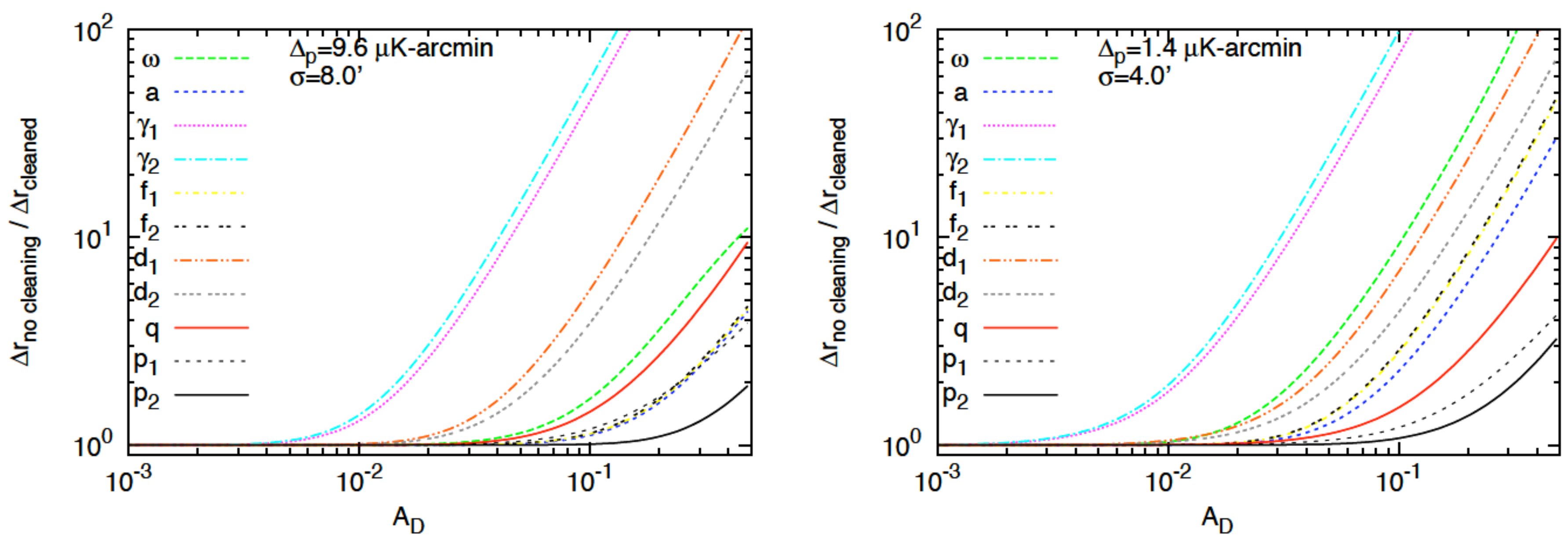
Yadav, Su, and Zaldarriaga (2010)

SELF CALIBRATING THE CMB

$$C_\ell^{BB}(cleaned) = \int \frac{d^2 \mathrm{l}_1}{(2\pi)^2} \left(W_Y^{\mathcal{D}}(\mathrm{l}_1, \mathrm{l}_2) \right)^2 \left[C_{\ell_1}^{XX} C_{\ell_2}^{\mathcal{DD}} - \frac{\left(C_{\ell_1}^{XX} C_{\ell_2}^{\mathcal{DD}} \right)^2}{\left(C_{\ell_2}^{XX} + \mathcal{N}_{\ell_2}^{XX} \right) \left(C_{\ell_2}^{\mathcal{DD}} + N_{\ell_2}^{\mathcal{DD}} \right)} \right]$$

$$\Delta r_{\text{no cleaning}} = \left[\frac{f_{sky}}{2} \sum_\ell (2\ell+1) \left(\frac{C_\ell^{BB}(\text{tensor})}{C_\ell^{BB}(\text{observed}) + \mathcal{N}_\ell^B} \right)^2 \right]^{-1/2}$$

$$\Delta r_{\text{with cleaning}} = \left[\frac{f_{sky}}{2} \sum_\ell (2\ell+1) \left(\frac{C_\ell^{BB}(\text{tensor})}{C_\ell^{BB}(\text{cleaned}) + \mathcal{N}_\ell^B} \right)^2 \right]^{-1/2}$$



Yadav, Su, and Zaldarriaga (2010)

ESTIMATING MULTIPLE DISTORTIONS SIMULTANEOUSLY

Most of the distortion estimators are orthogonal with low Correlations!

$$F_\ell^{\mathcal{D}\mathcal{D}'} = \int \frac{d^2 \mathbf{l}_1}{(2\pi)^2} f_{EB}^{\mathcal{D}}(\ell_1, \ell_2)(\mathbf{C}^{-1})_{\ell_1}^{EE} f_{EB}^{\mathcal{D}'}(\ell_1, \ell_2)(\mathbf{C}^{-1})_{\ell_2}^{BB}$$

$$\mathcal{C}_{qd_1} = .9$$

$$\mathcal{C}_{p2a} = -0.2$$

$$\mathcal{C}_{q\gamma_1} = .15$$

$$\mathcal{C}_{a\gamma_1} = .14$$

$$\langle \hat{\mathcal{D}}(L) \rangle_{CMB} = \mathcal{D}(L) + \frac{\sum_{\mathcal{D}'} F_L^{\mathcal{D}\mathcal{D}'} \mathcal{D}'(L)}{F_L^{\mathcal{D}\mathcal{D}}}$$

$$\mathcal{C}_{\mathcal{D}\mathcal{D}'} = \frac{F_\ell^{\mathcal{D}\mathcal{D}'}}{\sqrt{F_\ell^{\mathcal{D}\mathcal{D}} F_\ell^{\mathcal{D}'\mathcal{D}'}}}$$

Yadav, Su, and Zaldarriaga (2010)

Summary

* Weak lensing of the CMB very important for precision cosmology

  - potential confusion with primordial gravitational waves for r <~ 10^{-3}

* Instrumental systematic effects can not only produce artificial B-mode but also spurious projected lensing potential signal.

* Distortions can be imprinted on observed CMB by e.g. patchy reionization, Faraday rotation, cosmic strings etc. OR instrumental systematics which can produce both B-mode and lensing signal.

* Distortions on primary CMB can be detected by minimum variance estimators. B-mode produced by distortion fields can be found before they show up in B-mode power spectrum (self-diagnostics on primordial B-mode).


Thank you for
your attention!

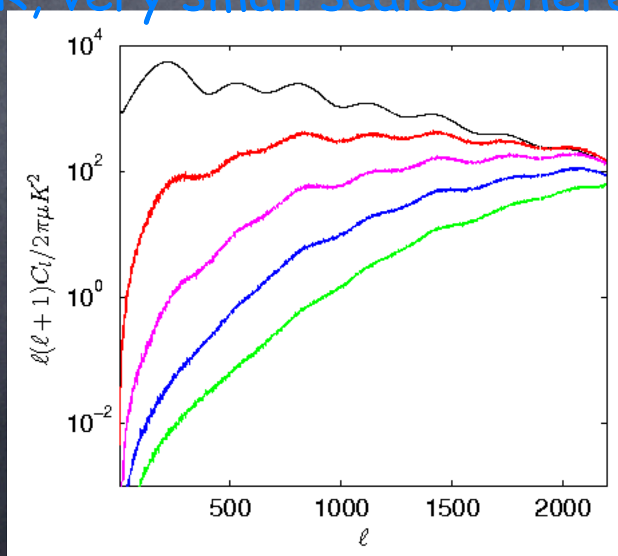
The limit of quadratic estimator

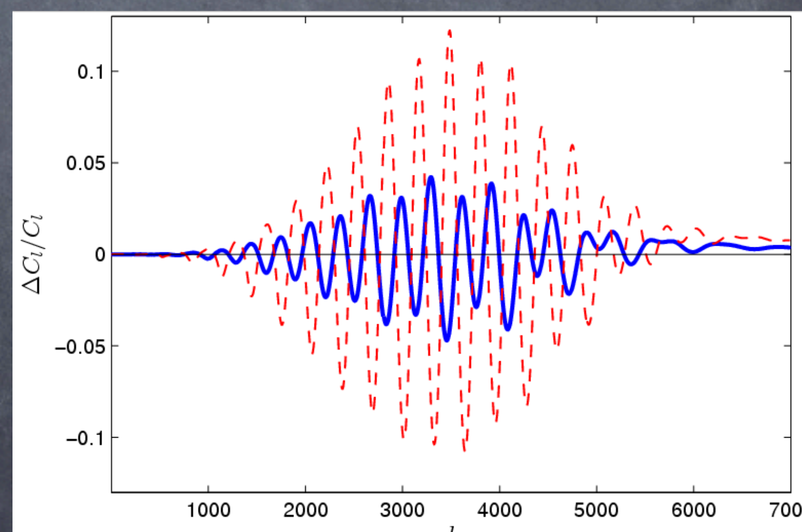
* Implemented in Fourier space - irregular map coverage becomes a problem
* Assuming uniform, uncorrelated noise, symmetrical beams
* Filtering treatment
* (Ignores higher order effects)
* Maxim likelihood method (not discussed here)

Series expansion in deflection angle?

$$\tilde{\Theta}(\mathrm{x}) = \Theta(\mathrm{x}') = \Theta(\mathrm{x} + \boldsymbol{\nabla}\psi)$$
$$\approx \Theta(\mathrm{x}) + \nabla^a\psi(\mathrm{x})\nabla_a\Theta(\mathrm{x}) + \tfrac{1}{2}\nabla^a\psi(\mathrm{x})\nabla^b\psi(\mathrm{x})\nabla_a\nabla_b\Theta(\mathrm{x}) + \ldots$$

Only a good approximation when:
- deflection angle much smaller than wavelength of temperature perturbation
- OR, very small scales where temperature is close to a gradient



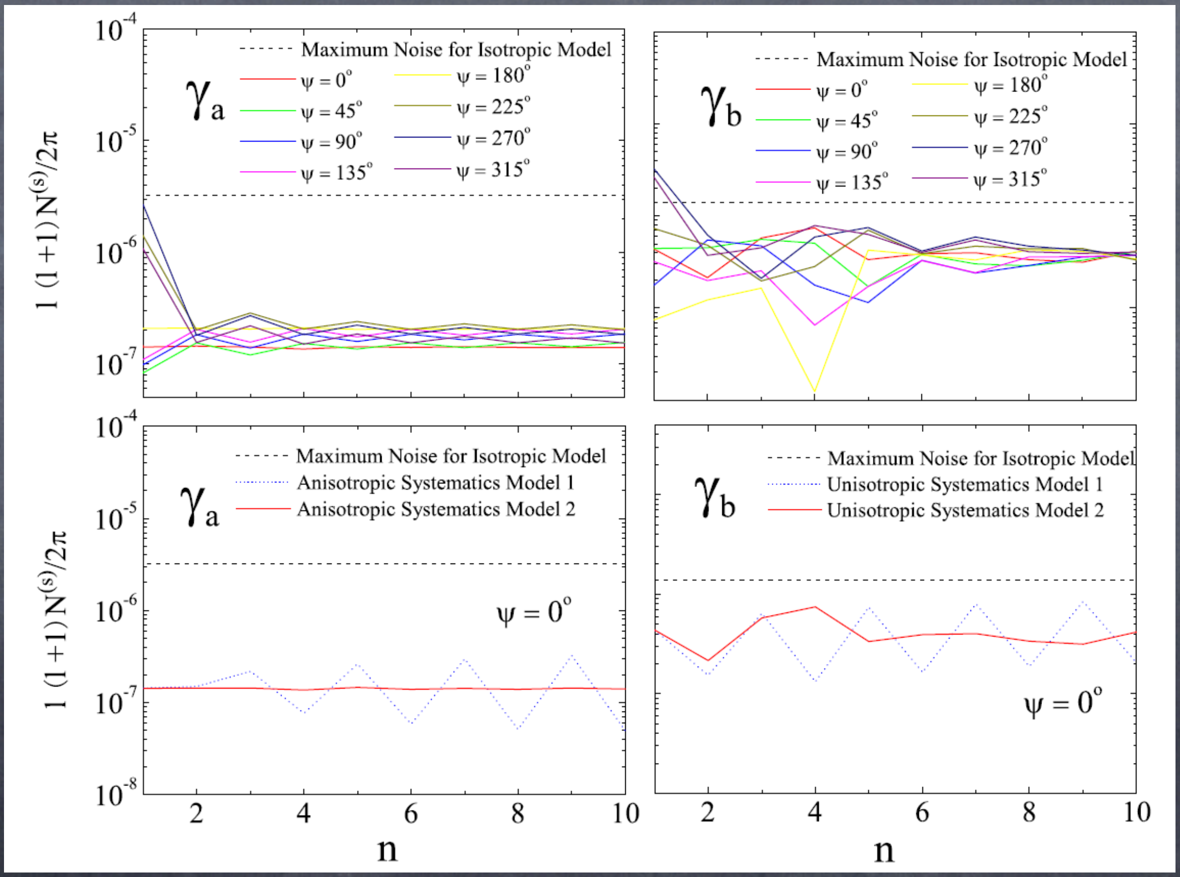


Series expansion only good on large and very small scales

A. Lewis, 2005

$$\alpha_S(l) = \tilde{\alpha}_S\{1 + B\cos^n(\phi_l + \psi)\}, \qquad \text{Model 1}$$

$$\alpha_S(l) = \tilde{\alpha}_S\{1 + B\cos(n\phi_l + \psi)\}, \qquad \text{Model 2}$$



How we did the simulations

$$\hat{\tau}_{\hat{\mathbf{l}}}^{EB} = -N_l^{EB} \int d^2\hat{\mathbf{n}} e^{-i\hat{\mathbf{n}}\cdot\hat{\mathbf{l}}} \mathrm{Re}\left\{\left[\mathbf{G}^{EB}(\hat{\mathbf{n}}) L^{B*}(\hat{\mathbf{n}})\right]\right\}$$

$$\mathbf{G}_{\hat{\mathbf{l}}}^{EB} = \frac{C^{EE}}{(C_l^{EE} + N_l^{EE})} E(\hat{\mathbf{l}}) e^{2i\varphi_{\hat{\mathbf{l}}}}$$

$$L_{\hat{\mathbf{l}}}^{B} = \frac{B(\hat{\mathbf{l}})}{(C_l^{BB} + N_l^{BB})} e^{2i\varphi_{\hat{\mathbf{l}}}}$$

Hu et al. (2007)