Full-sky CMB lensing reconstruction in presence of galactic residuals and point sources

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Estimation of the lensing potential from full-sky maps

✓ Full-sky quadratic estimator

$$\hat{\phi}_L^M \propto A_L \int d\hat{\mathbf{n}} Y_L^{M*} \left(\sum_{\ell_1 m_1} \frac{1}{C_{\ell_1}^{\rm tot}} \Theta_{\ell_1}^{m_1} Y_{\ell_1}^{m_1} \right) \nabla \left(\sum_{\ell_2 m_2} \frac{\tilde{C}_{\ell_2}}{C_{\ell_2}^{\rm tot}} \Theta_{\ell_2}^{m_2} Y_{\ell_2}^{m_2} \right)$$

✓ Estimator is unbiased

Okamoto & Hu (2003)

$$\langle \hat{\phi}_L^M \rangle|_{\rm lens} = \phi_L^M$$

✓ Estimator variance

$$\langle \hat{\phi}_L^M \hat{\phi}_L^{M*} \rangle = C_L^{\phi\phi} + N_L^{(0)} + N_L^{(1)} + N_L^{(2)} + \cdots$$

Gaussian noise

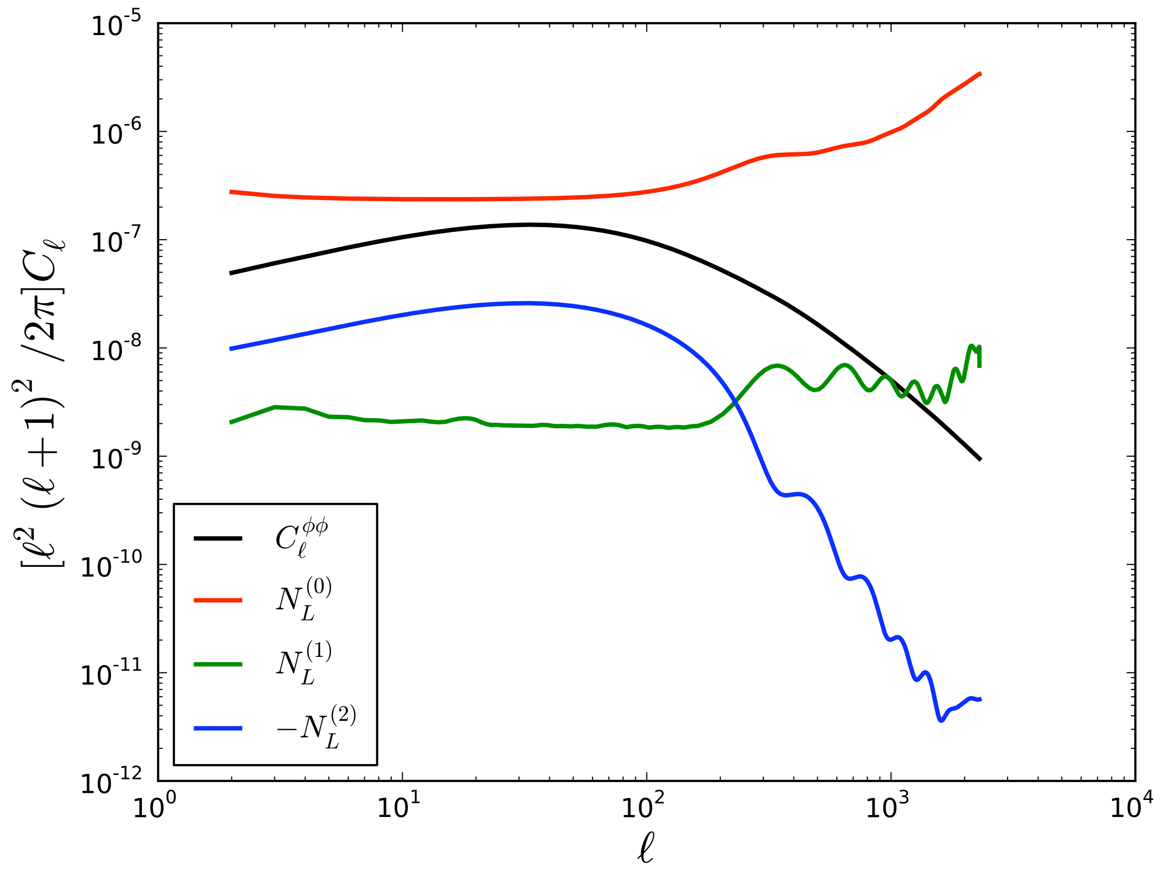
high order biases

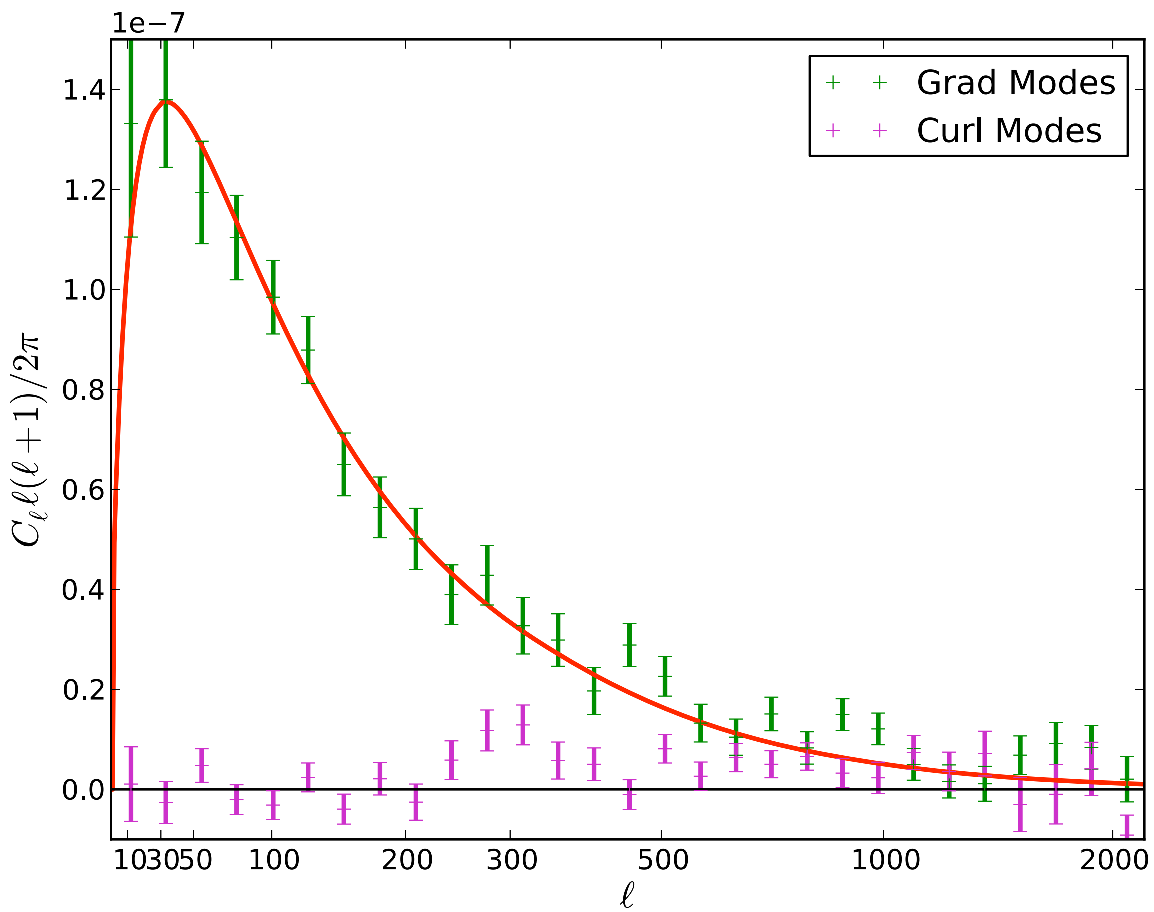
Kesden *et al.* (2003), Hanson *et al.* (2011)

Estimation of the lensing potential from full-sky maps

✓ Reconstruction in a perfect pessimistic Planck-like case:

- noise = 60μk.arcmin
- 5 arcmin beam
- l_{max}=2300



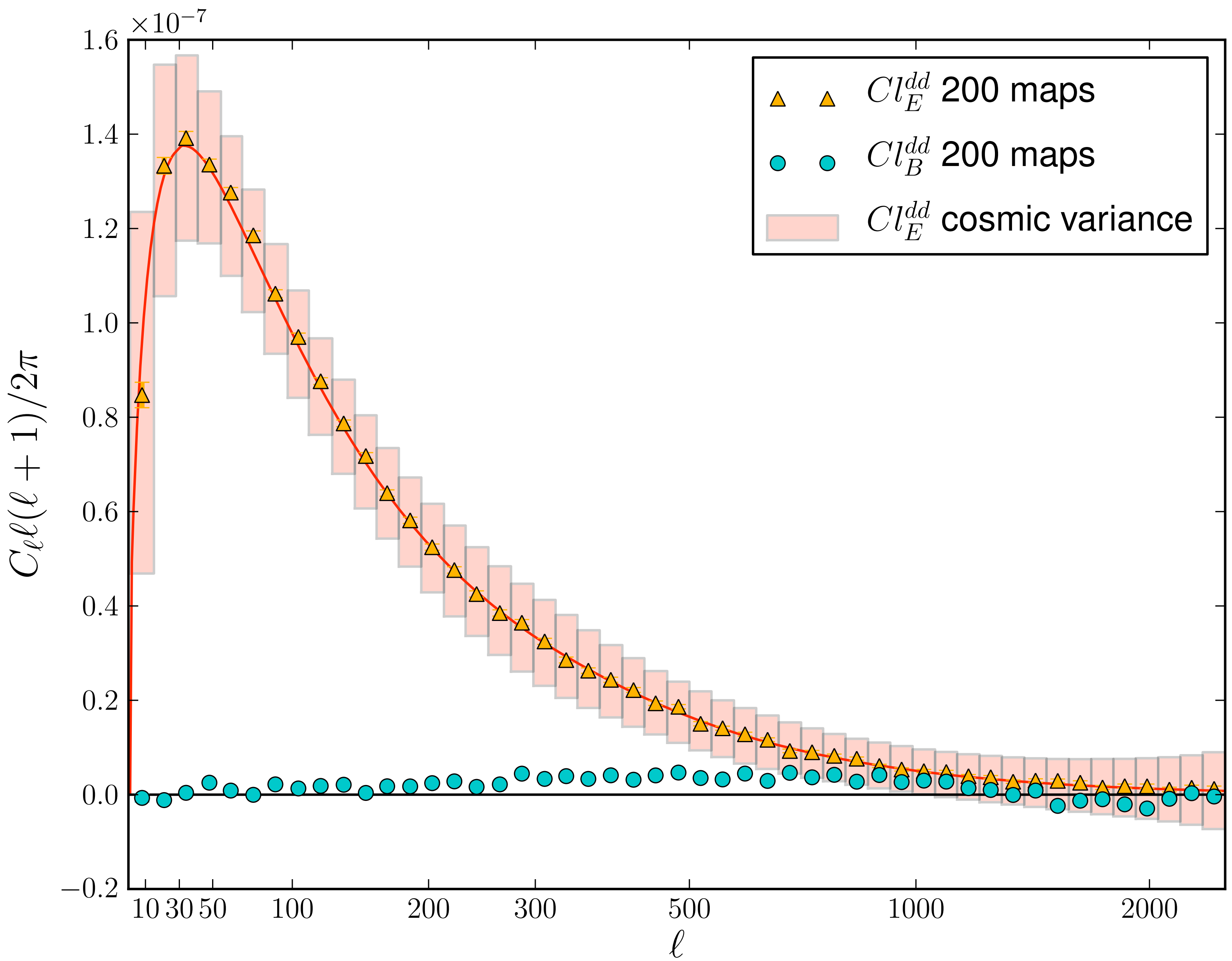


Perfect simulation:

- Homogeneous coverage
- White and known noise
- No sky cuts
- Known cosmology

Estimation of the lensing potential from full-sky maps

Lensing code: iLens   Basak *et al.* 2009

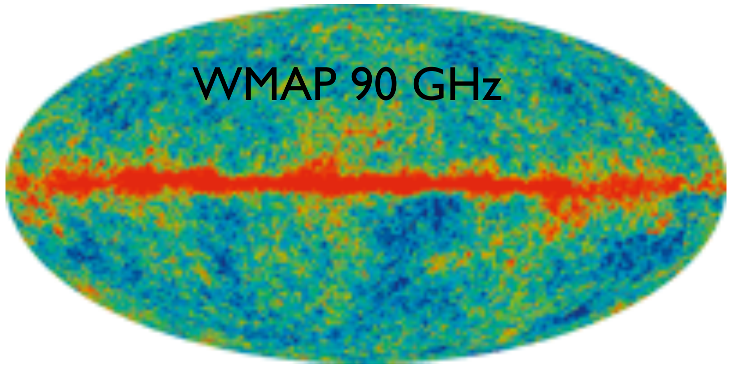


- Unbiased reconstruction of the lensing potential
- Non-zero curl modes

Complications

✓ Galaxy

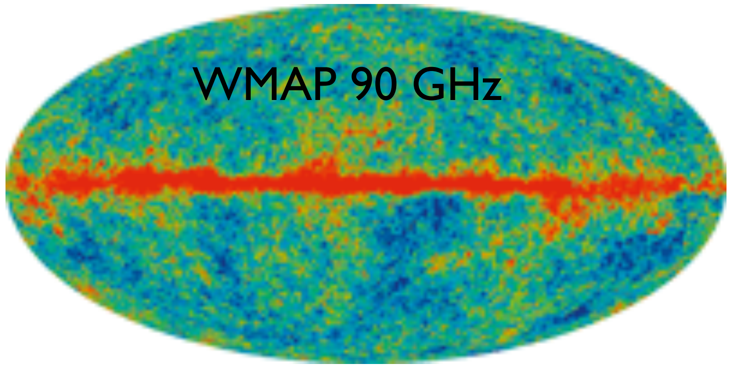
- Component separation will help but residuals expected
- Masking large regions of the sky is necessary



✓ How to handle galactic mask ?

- Analytically ?

Complications



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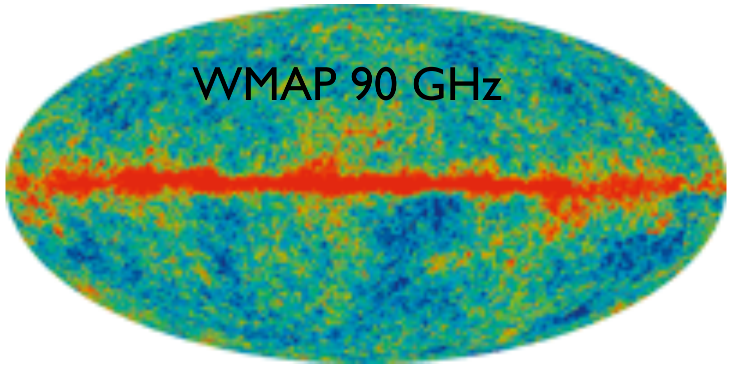
$$\langle \hat{\phi}_L^M \hat{\phi}_L^{M*} \rangle = \sum_\lambda K_\lambda^{LM} C_\lambda^{\phi\phi} + N_L^{(0)\prime} + \cdots$$

$$\begin{aligned} K_\lambda^{LM} &= \frac{1}{2^7} \int d^7 \cos\theta_i \;\; \lambda(\lambda+1) \frac{\Pi_\lambda}{4\pi} d^\lambda_{11}(\theta_7) \left(\frac{\Pi_L}{\sqrt{4\pi}} (-1)^M d^L_{-M1}(\theta_1) \right) \left(\frac{\Pi_L}{\sqrt{4\pi}} (-1)^M d^L_{-M1}(\theta_2) \right) \times \\ & \left(\sum_{l_1 m_1} \Pi^2_{l_1} f_1(l_1) d^{l_1}_{m_1 0}(\theta_1) d^{l_1}_{m_1 0}(\theta_3) \right) \left(\sum_{l_3 m_3} \Pi^2_{l_3} f_1(l_3) d^{l_3}_{m_3 0}(\theta_2) d^{l_3}_{m_3 0}(\theta_5) \right) \times \\ & \left(\sum_{l_2 m_2} \Pi^2_{l_2} \sqrt{l_2(l_2+1)} f_2(l_2) d^{l_2}_{m_2 -1}(\theta_1) d^{l_2}_{m_2 0}(\theta_4) \right) \left(\sum_{l_4 m_4} \Pi^2_{l_4} \sqrt{l_4(l_4+1)} f_2(l_4) d^{l_4}_{m_4 -1}(\theta_2) d^{l_4}_{m_4 0}(\theta_6) \right) \times \\ & \left(\sum_{l'_1 m'_1} C^{\theta\theta}_{l'_1} \Pi^2_{l'_1} d^{l'_1}_{-m'_1 0}(\theta_3) d^{l'_1}_{-m'_1 0}(\theta_5) \right) \left(\sum_{l'_2 m'_2} \Pi^2_{l'_2} d^{l'_2}_{-m'_2 0}(\theta_4) d^{l'_2}_{-m'_2 0}(\theta_6) d^{l'_2}_{00}(\theta_7) \right) \left(\sum_{\lambda'_2} C^{\theta\theta}_{\lambda'_2} \lambda'_2(\lambda'_2+1) \frac{\Pi^2_{\lambda'_2}}{4\pi} d^{\lambda'_2}_{-1-1}(\theta_7) \right) \times \\ & \left(\sum_{l''_1 m''_1} w^*_{l''_1 m''_1} \Pi_{l''_1} d^{l''_1}_{m''_1 0}(\theta_3) \right) \left(\sum_{l''_2 m''_2} w^*_{l''_2 m''_2} \Pi_{l''_2} d^{l''_2}_{m''_2 0}(\theta_4) \right) \left(\sum_{l''_3 m''_3} w_{l''_3 m''_3} \Pi_{l''_3} d^{l''_3}_{m''_3 0}(\theta_5) \right) \left(\sum_{l''_4 m''_4} w_{l''_4 m''_4} \Pi_{l''_4} d^{l''_4}_{m''_4 0}(\theta_6) \right) \end{aligned}$$

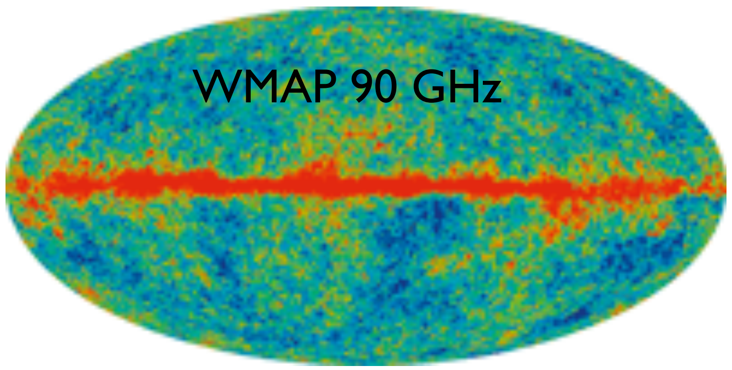
Benoit-Lévy *et al., in prep*

Complications



- ✓ Galaxy
    - Component separation will help but residuals expected
    - Masking large regions of the sky is necessary
- ✓ How to handle galactic mask ?
    - Analytically ?
    - Inverse covariance weighting is difficult
        - Planck resolution
        - Low noise, large dynamic
    - Inpainting technics hard to control
- ✓ We want a simple, robust and linear pipeline

Complications



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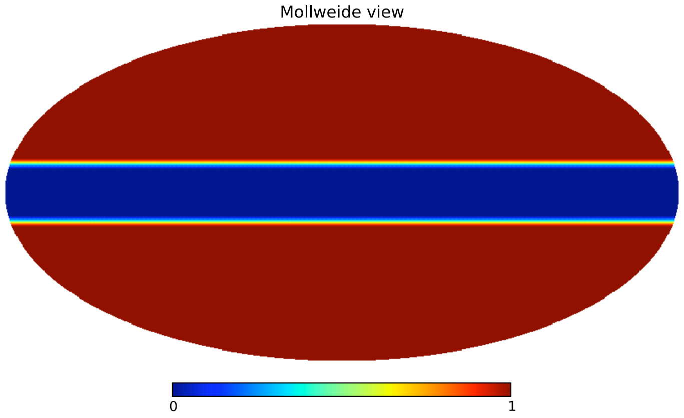
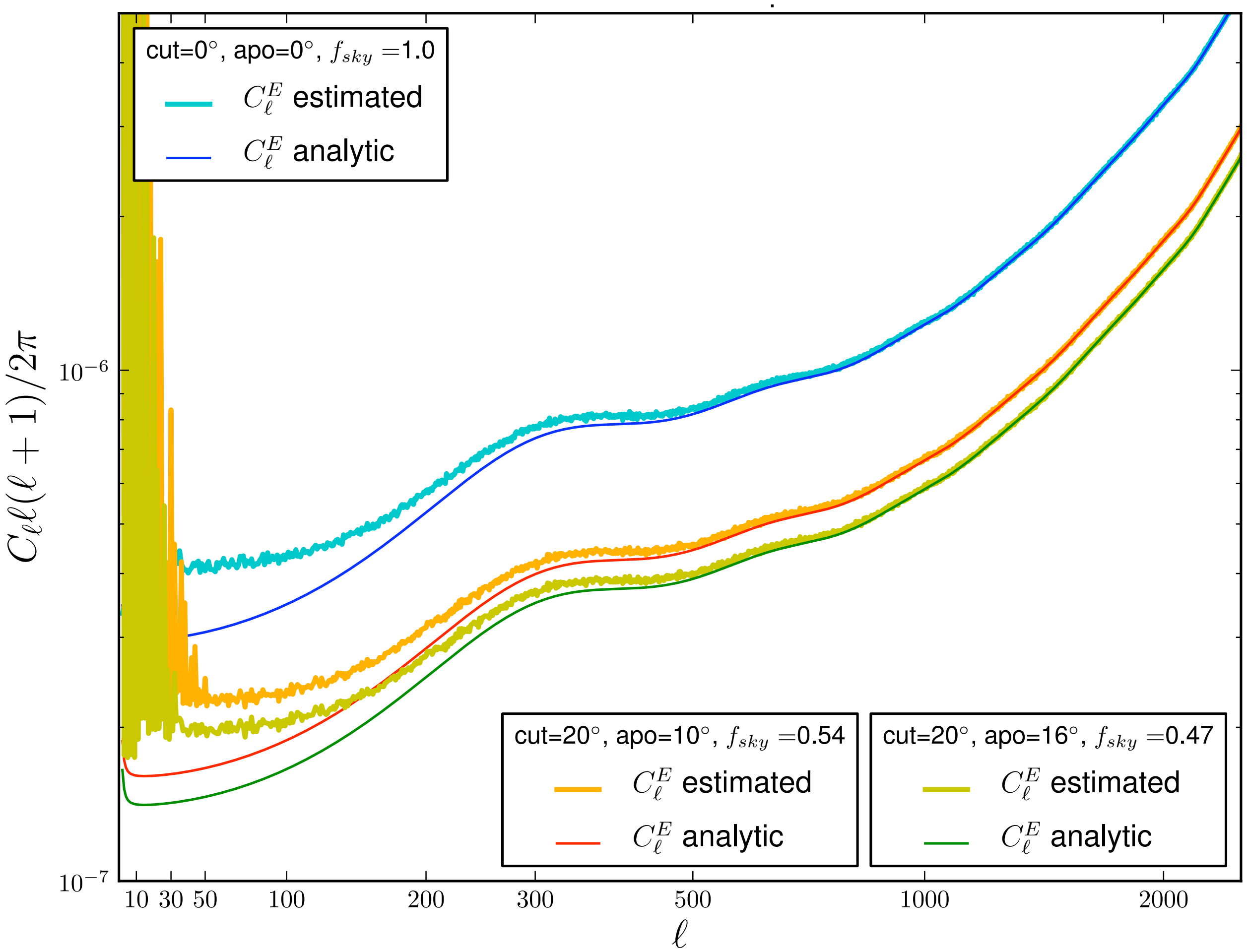
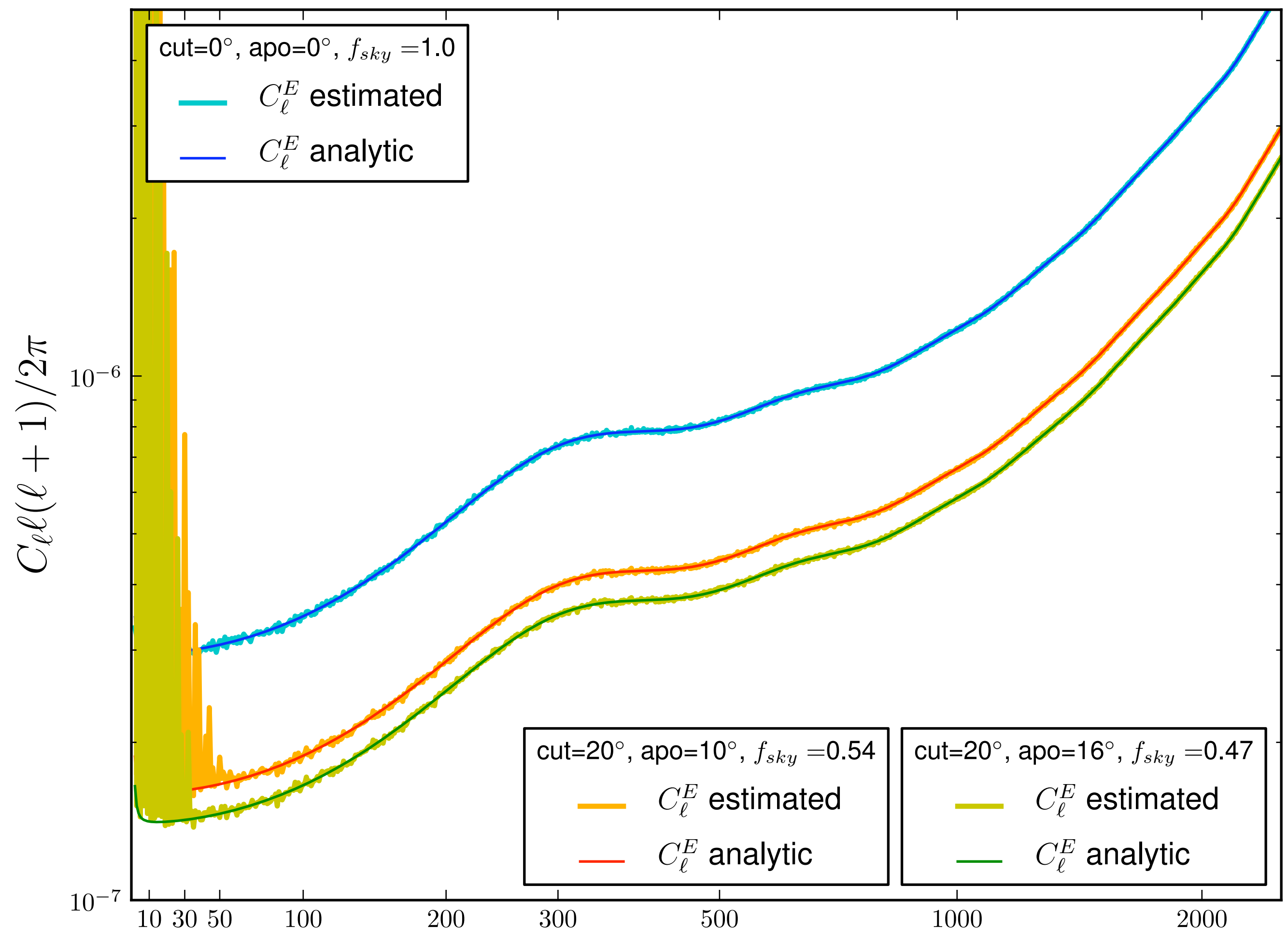
- Apodized galactic cut



Apodized galactic mask



$$f_{sky} = \frac{1}{N_{pix}} \sum_i w_i^4$$



Analytical treatment of masks

Azimuthal mask: some simplifications...

$$\langle \hat{\phi}_L^M \hat{\phi}_L^{M*} \rangle = \sum_\lambda K_\lambda^{LM} C_\lambda^{\phi\phi} + N_L^{(0)\prime} + \cdots$$

$$K_\lambda^{LM} = \frac{1}{2^7} \int d^7 \cos\theta_i \;\; \lambda(\lambda+1) \frac{\Pi_\lambda}{4\pi} d_{11}^\lambda(\theta_7) \left(\frac{\Pi_L}{\sqrt{4\pi}} d_{-M1}^L(\theta_1) \right) \left(\frac{\Pi_L}{\sqrt{4\pi}} d_{-M1}^L(\theta_2) \right) \left(\sum_{\lambda_2'} C_{\lambda_2'}^{\theta\theta} \lambda_2'(\lambda_2'+1) \frac{\Pi_{\lambda_2'}^2}{4\pi} d_{-1-1}^{\lambda_2'}(\theta_7) \right) \times$$

$$\left(\sum_{l_1 m_1 l_3 l_1'} C_{l_1'}^{\theta\theta} \Pi_{l_1 l_3 l_1'}^2 f_1(l_1) f_1(l_3) d_{m_1 0}^{l_1}(\theta_1) d_{m_1 0}^{l_1}(\theta_3) d_{m_1 0}^{l_3}(\theta_2) d_{m_1 0}^{l_3}(\theta_5) d_{-m_1 0}^{l_1'}(\theta_3) d_{-m_1 0}^{l_1'}(\theta_5) \right) \times$$

$$\left(\sum_{l_2 m_2 l_4 l_2'} \Pi_{l_2 l_4 l_2'}^2 \sqrt{l_2(l_2+1)} f_2(l_2) \sqrt{l_4(l_4+1)} f_2(l_4) d_{m_2 -1}^{l_2}(\theta_1) d_{m_2 0}^{l_2}(\theta_4) d_{m_2 -1}^{l_4}(\theta_2) d_{m_2 0}^{l_4}(\theta_6) d_{-m_2 0}^{l_2'}(\theta_4) d_{-m_2 0}^{l_2'}(\theta_6) d_{00}^{l_2'}(\theta_7) \right) \times$$

$$\left(\sum_{l_1''} w_{l_1'' 0}^* \Pi_{l_1''} d_{00}^{l_1''}(\theta_3) \right) \left(\sum_{l_2''} w_{l_2'' 0}^* \Pi_{l_2''} d_{00}^{l_2''}(\theta_4) \right) \left(\sum_{l_3''} w_{l_3'' 0} \Pi_{l_3''} d_{00}^{l_3''}(\theta_5) \right) \left(\sum_{l_4''} w_{l_4'' 0} \Pi_{l_4''} d_{00}^{l_4''}(\theta_6) \right)$$

Benoit-Lévy *et al.*, *in prep*

$N^{(0)}$ is (a bit) simpler: only 6d integral

$$\langle \hat{\phi}_L^M \hat{\phi}_L^{M*} \rangle = C_L^{\phi\phi} + N_L^{(0)} + N_L^{(1)} + N_L^{(2)} + \cdots$$

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& \left(\sum_{l_1 m_1 l_3 l'_1} C^{\theta\theta}_{l'_1}\Pi^2_{l_1 l_3 l'_1} f_1(l_1) f_1(l_3) d^{l_1}_{m_1 0}(\theta_1) d^{l_1}_{m_1 0}(\theta_3) d^{l_3}_{m_1 0}(\theta_2) d^{l_3}_{m_1 0}(\theta_5) d^{l'_1}_{-m_1 0}(\theta_3) d^{l'_1}_{-m_1 0}(\theta_5)\right)\times \\
& \left(\sum_{l_2 m_2 l_4 l'_2} \Pi^2_{l_2 l_4 l'_2}\sqrt{l_2(l_2+1)} f_2(l_2)\sqrt{l_4(l_4+1)} f_2(l_4) d^{l_2}_{m_2 -1}(\theta_1) d^{l_2}_{m_2 0}(\theta_4) d^{l_4}_{m_2 -1}(\theta_2) d^{l_4}_{m_2 0}(\theta_6) d^{l'_2}_{-m_2 0}(\theta_4) d^{l'_2}_{-m_2 0}(\theta_6) d^{l'_2}_{00}(\theta_7)\right)\times \\
& \left(\sum_{l''_1} w^*_{l''_1 0}\Pi_{l''_1} d^{l''_1}_{00}(\theta_3)\right)\left(\sum_{l''_2} w^*_{l''_2 0}\Pi_{l''_2} d^{l''_2}_{00}(\theta_4)\right)\left(\sum_{l''_3} w_{l''_3 0}\Pi_{l''_3} d^{l''_3}_{00}(\theta_5)\right)\left(\sum_{l''_4} w_{l''_4 0}\Pi_{l''_4} d^{l''_4}_{00}(\theta_6)\right)
\end{aligned}$$

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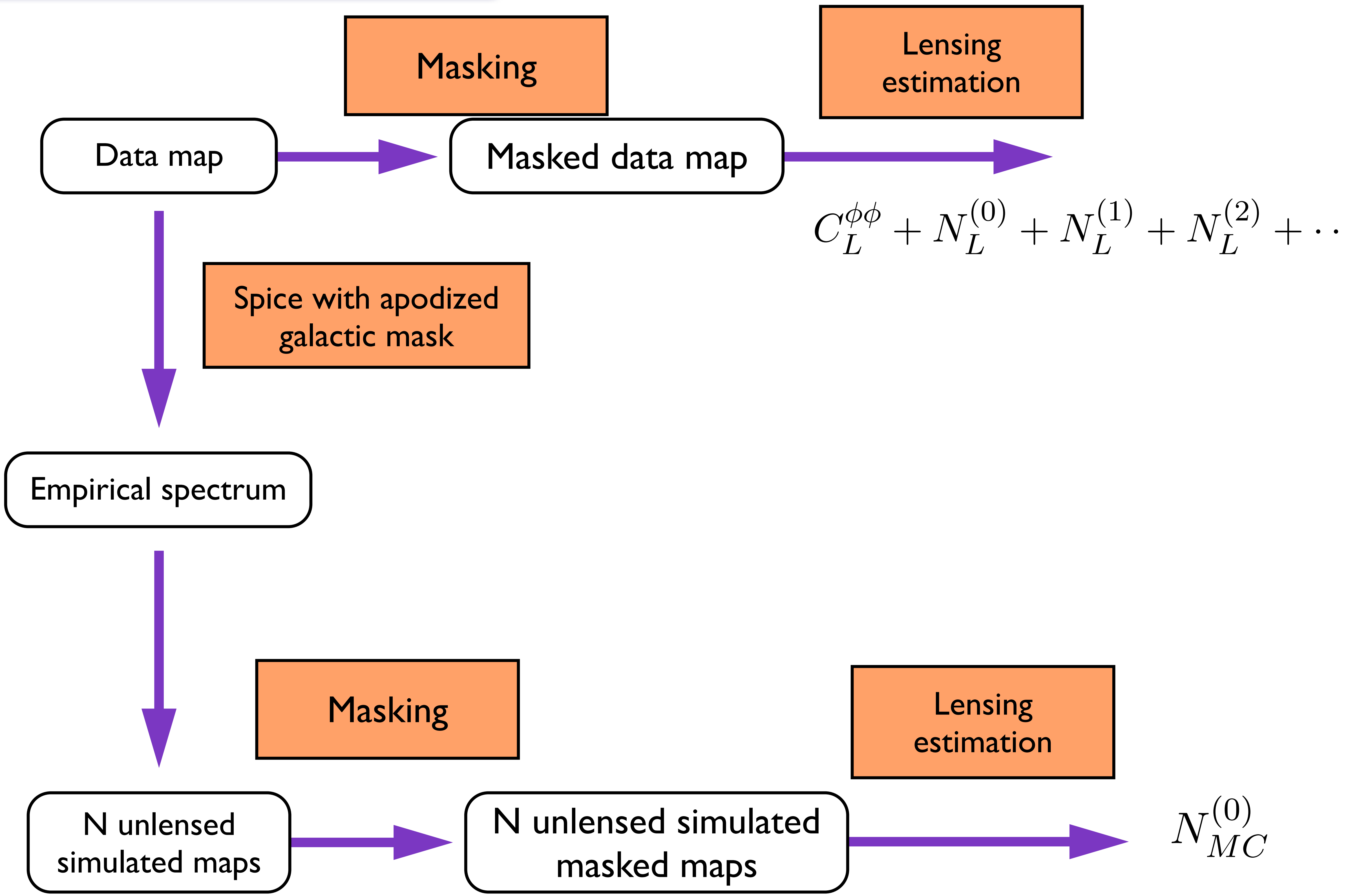
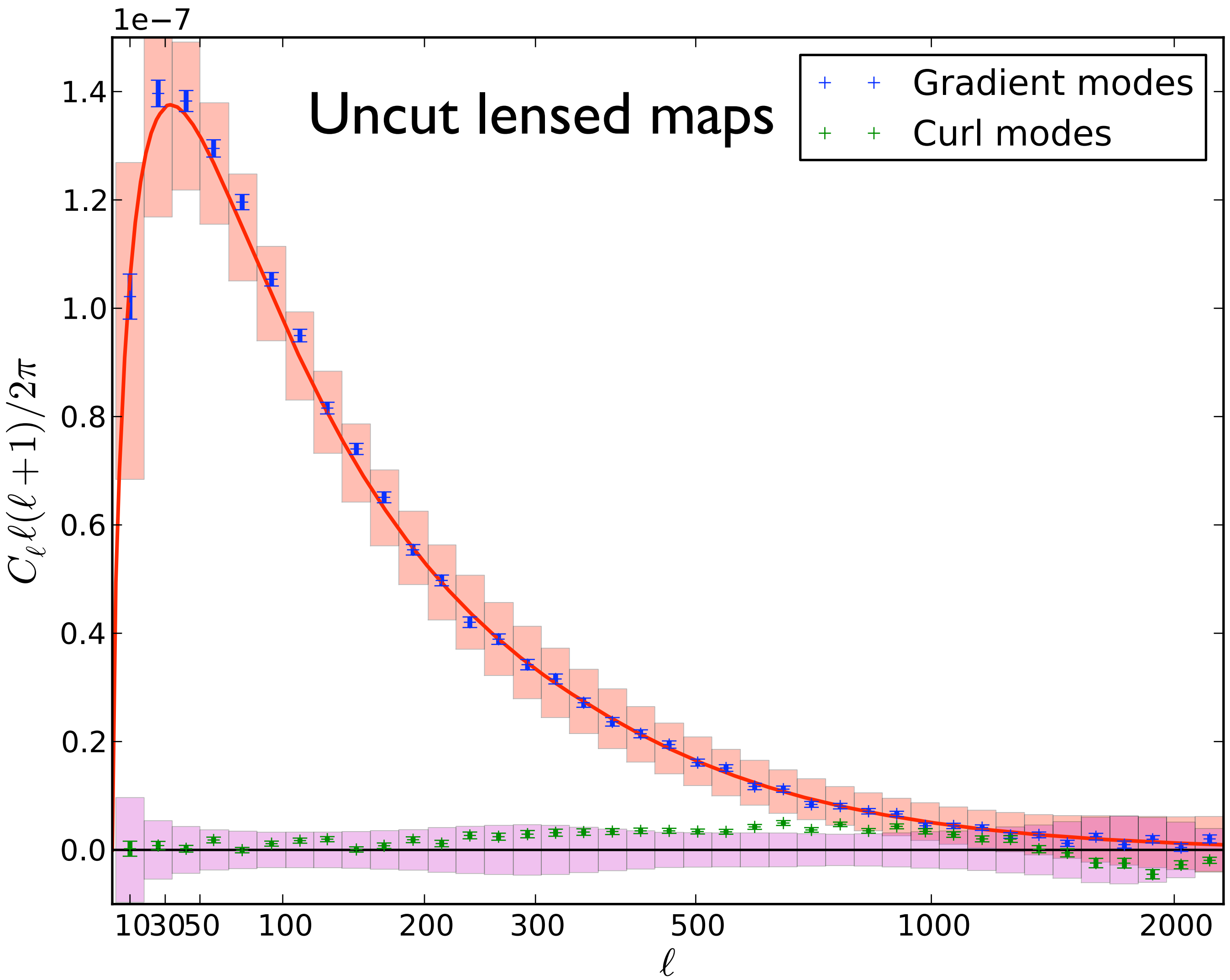
Applying the estimator on unlensed maps gives N_0: computation of the bias by Monte-Carlo

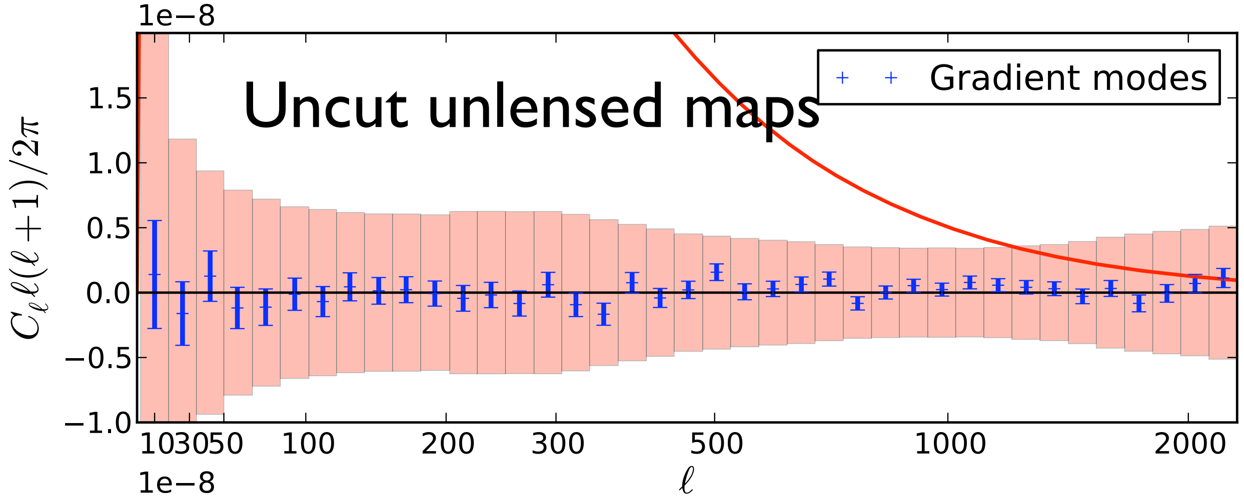
Pipeline description

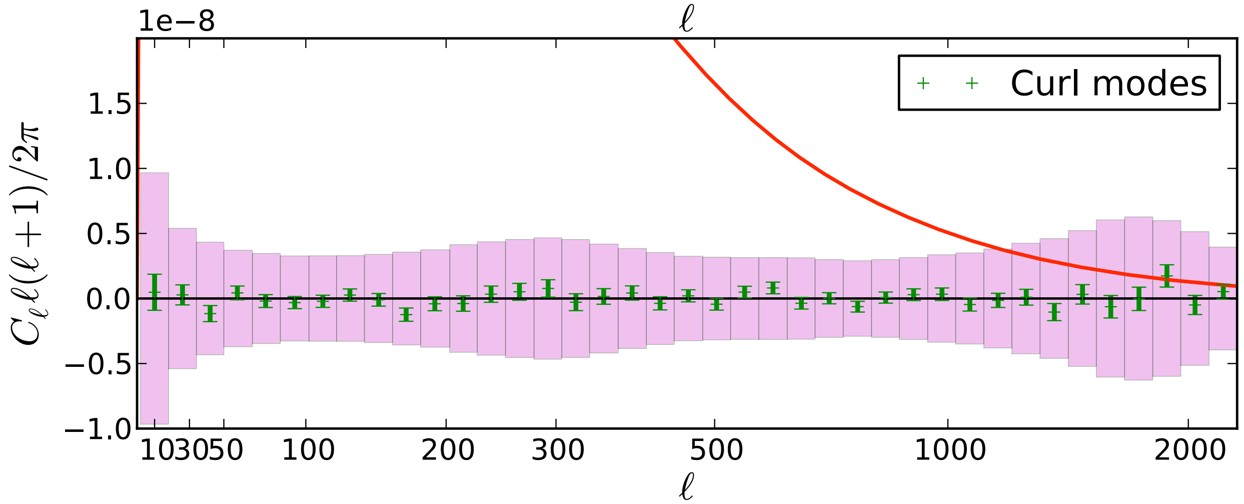


Null-tests of the pipeline

Tests are made on 50 'data' maps

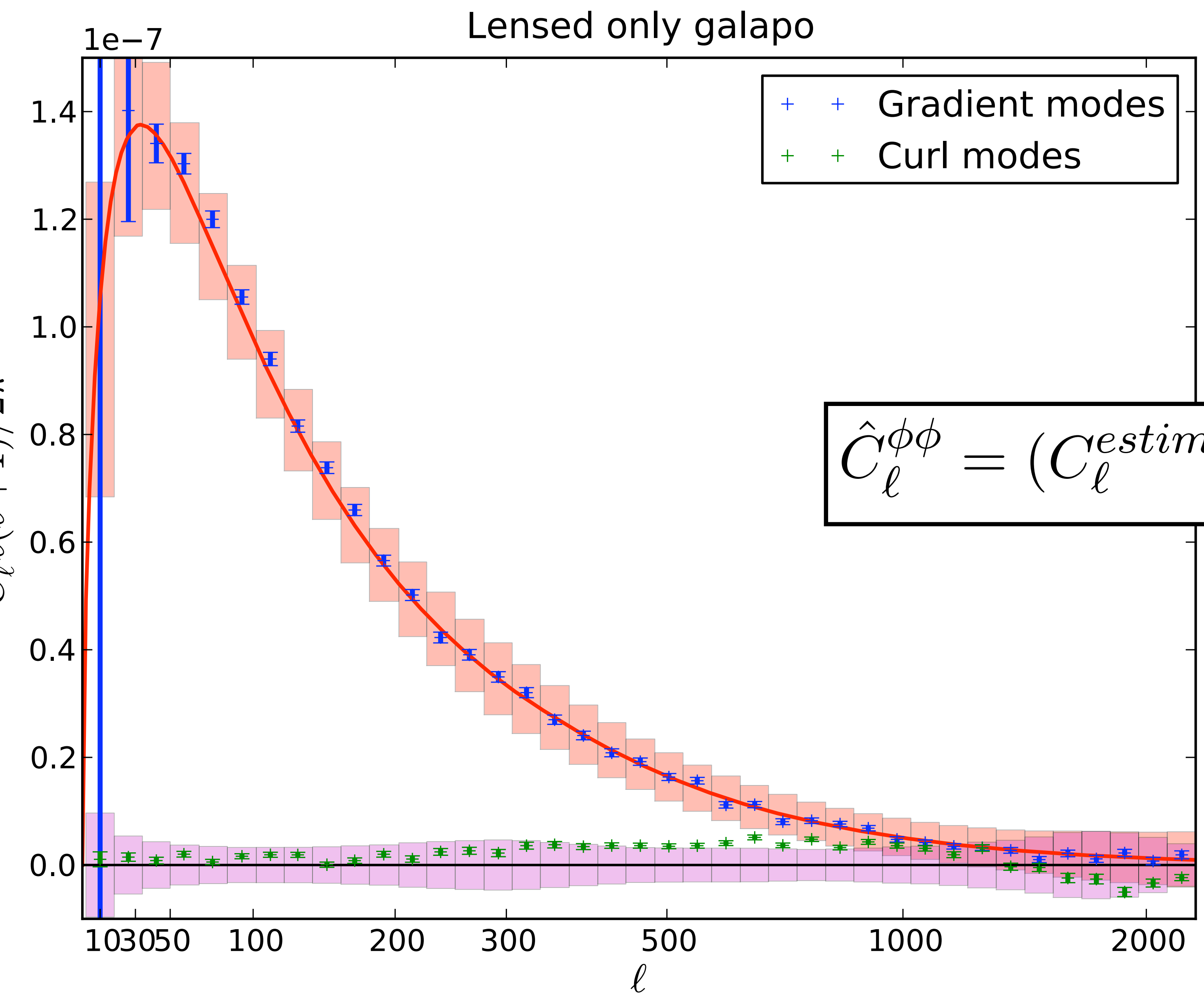






Using empirical spectrum from the data do not induce bias

Lensing potential reconstruction



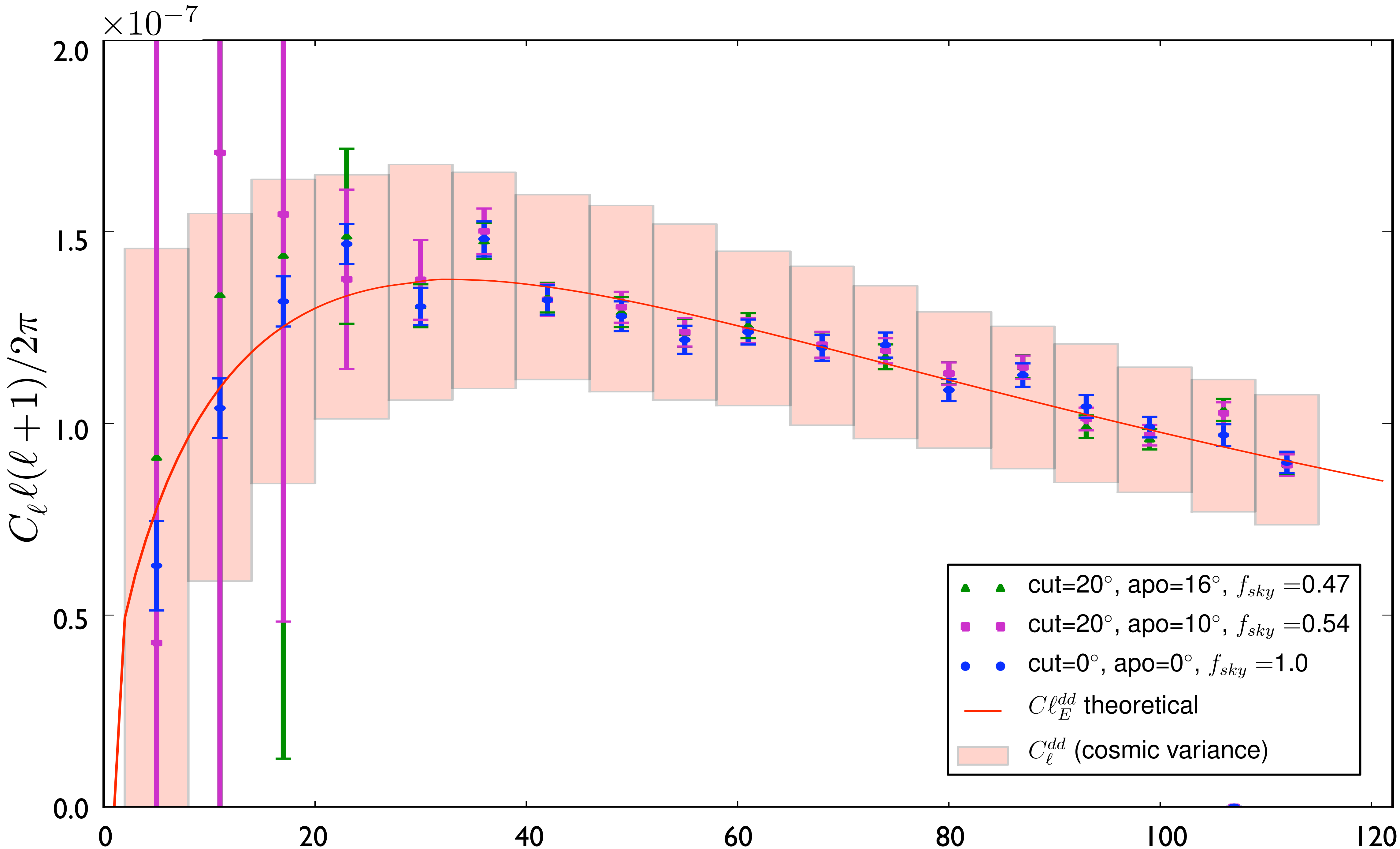
$$\hat{C}_\ell^{\phi\phi} = (C_\ell^{estim} - N_{MC}^0)/f_{gal} - N_{th}^{(1)} - N_{th}^{(2)}$$

Apodized galactic mask allows for unbiased reconstruction

Loss of large scale-modes

Coupling matrix close to identity. Band covariances under study

Apodization parameter dependance



Large apodization length increases missing fraction but reduces contamination

Point sources

Inpainting by local constrained Gaussian realizations

Aim : fill the holes with pure Gaussian CMB

$$T = \begin{pmatrix} T_1 \\ T_2 \end{pmatrix}$$

Hole (T_1)

Constraint (T_2)

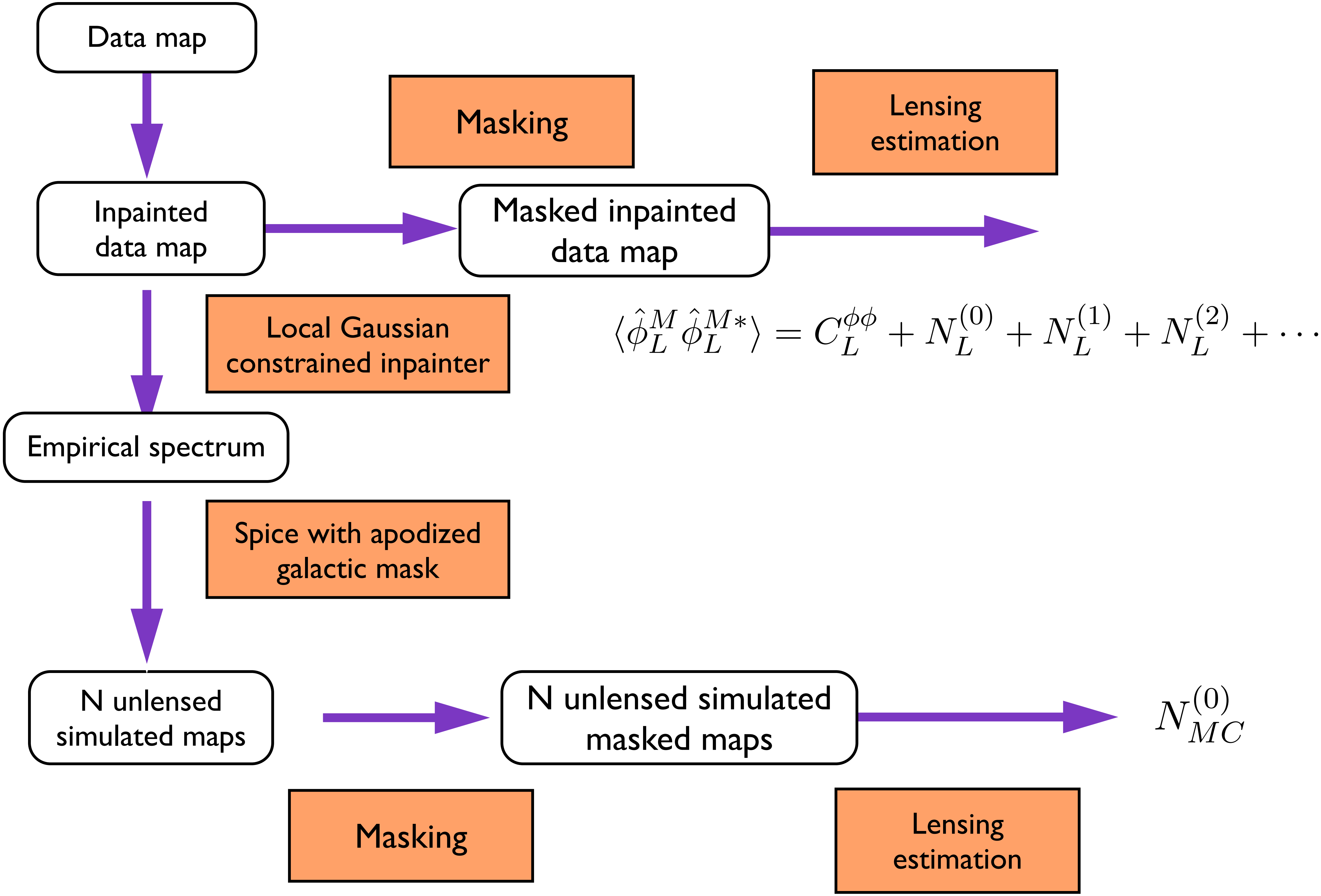
$$P(T_1|T_2) = \mathcal{N}(C_{12}C_{22}^{-1}T_2, C_{11} - C_{12}C_{22}^{-1}C_{21})$$

$$T_1 = \tilde{T}_1 + C_{12}C_{22}^{-1}(T_2 - \tilde{T}_2)$$

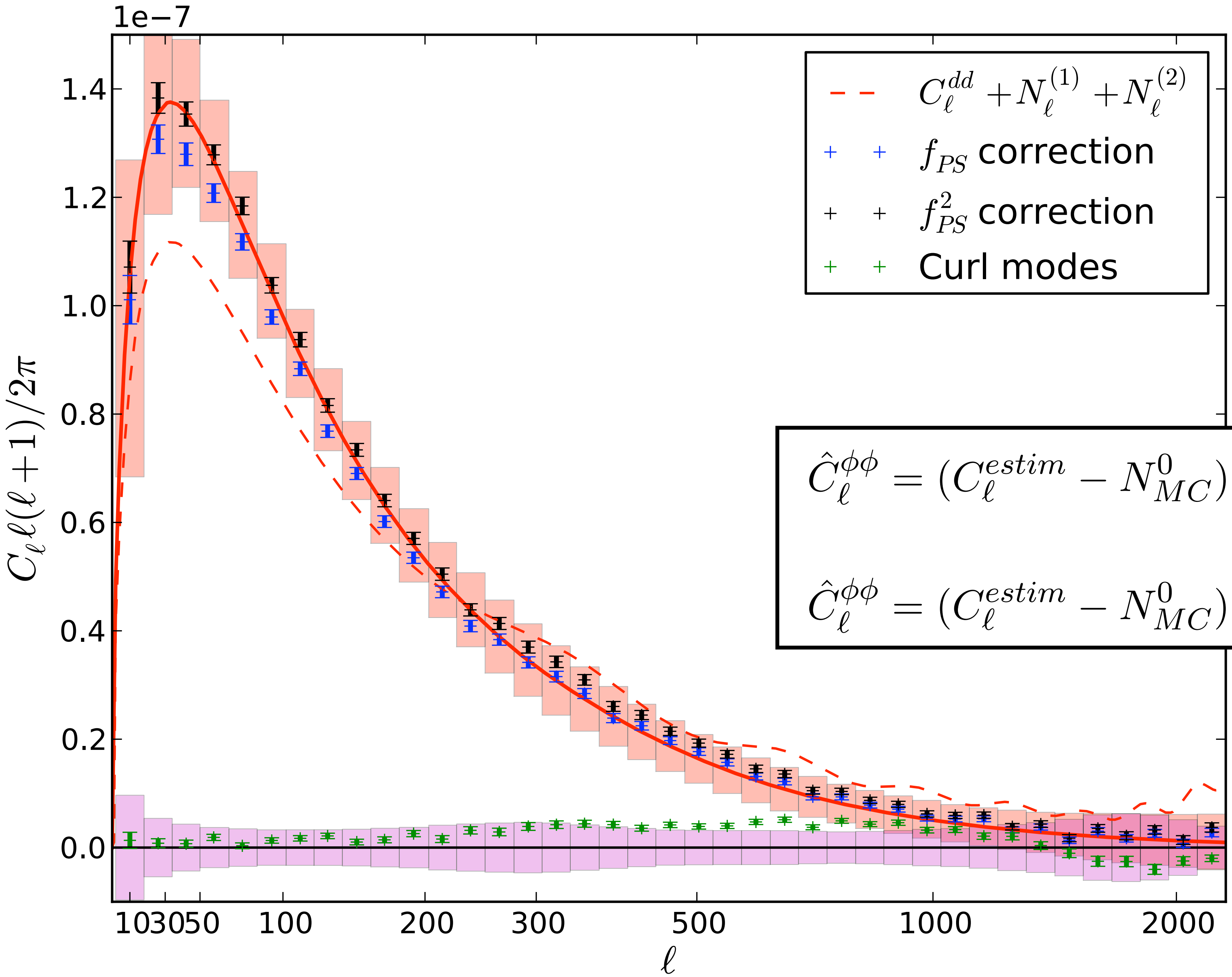
Simulated Gaussian CMB ($\tilde{T}_1$, $\tilde{T}_2$)

We expect a loss of power due to missing lensing in inpainted holes

Pipeline description



Only point source inpainting



$$\hat{C}_\ell^{\phi\phi} = (C_\ell^{estim} - N_{MC}^0)/f_{PS} - N_{th}^{(1)} - N_{th}^{(2)}$$

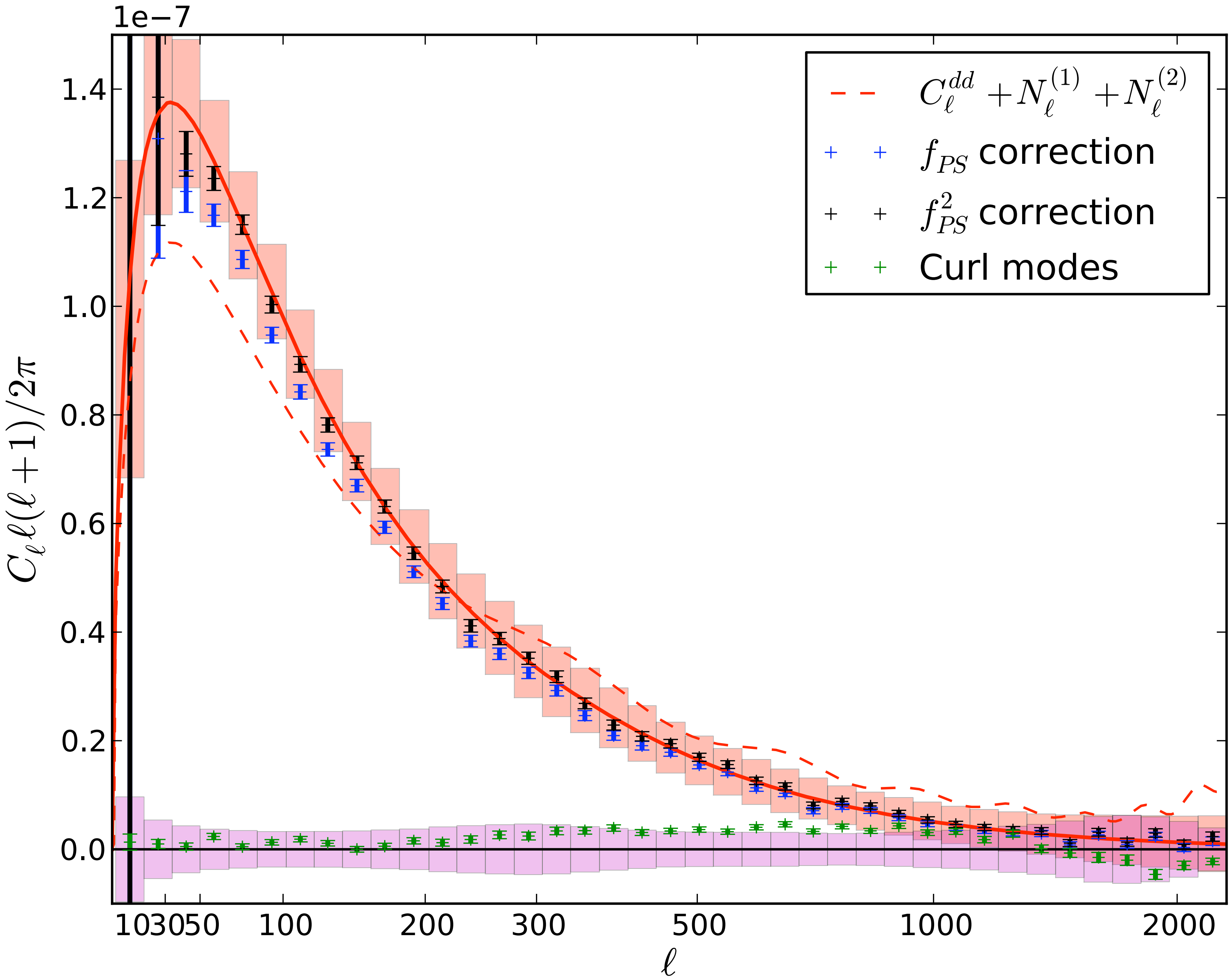
$$\hat{C}_\ell^{\phi\phi} = (C_\ell^{estim} - N_{MC}^0)/f_{PS}^2 - N_{th}^{(1)} - N_{th}^{(2)}$$

As Gaussian CMB is restored, one would expect a correction by a factor f_{PS}, not f_{PS}^2

Reconstruction within theoretical error bars

Modes coupling become non-trivial

Galactic mask and point sources inpainting



Reconstruction within theoretical error bars

Conclusions

✓ Dealing with galaxy in full-sky lensing recosntruction

- Dealing with galaxy can be done using a simple apodized mask
- Unbiased reconstruction but loss of precision at large scales

✓ Gaussian inpainting of resolved point sources recovers most of the lensing power

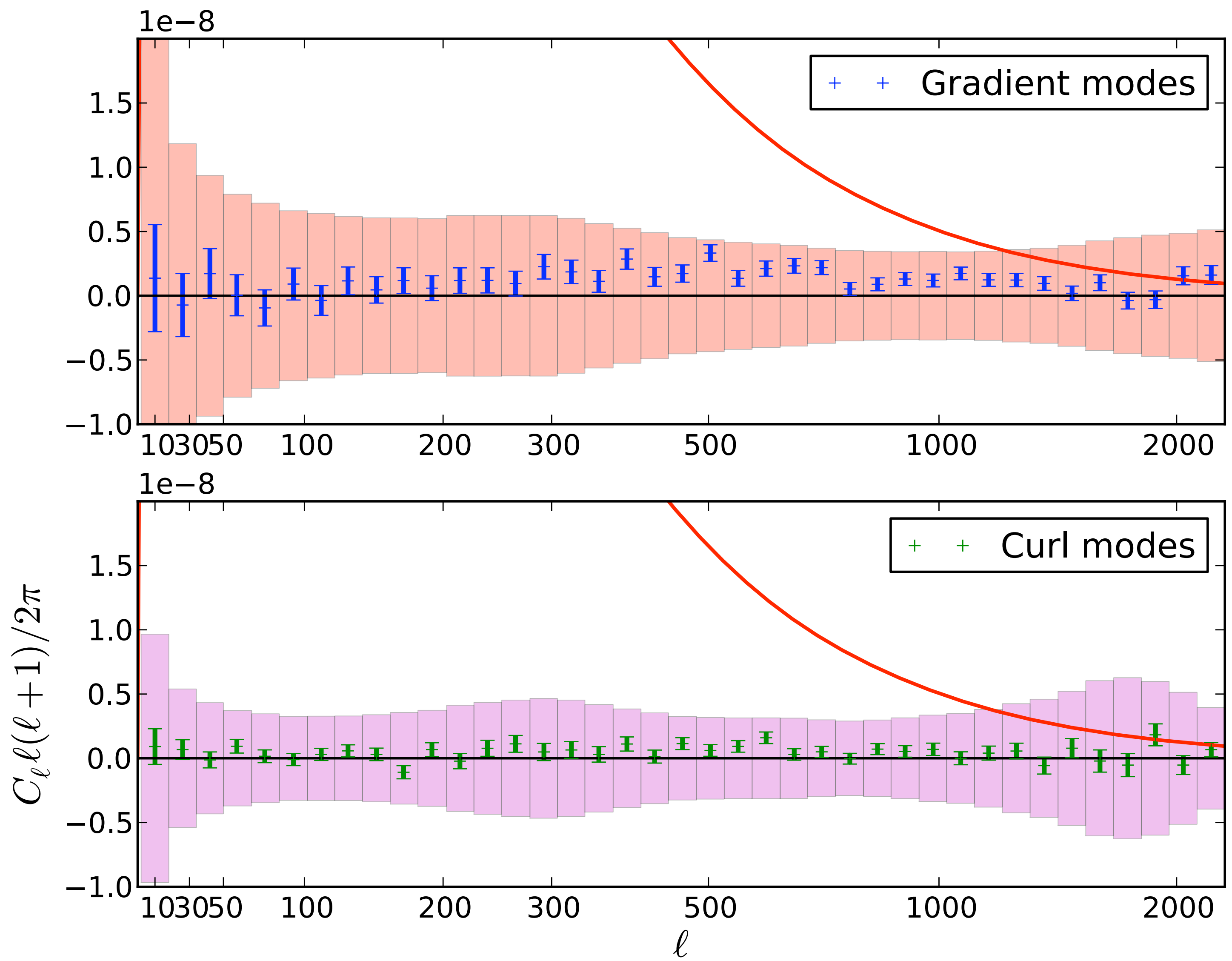
- Non-trivial mode coupling
- Effect on $N^{(1)}$ and $N^{(2)}$

✓ Under study : error bars correlations due to modes coupling

Thank you!

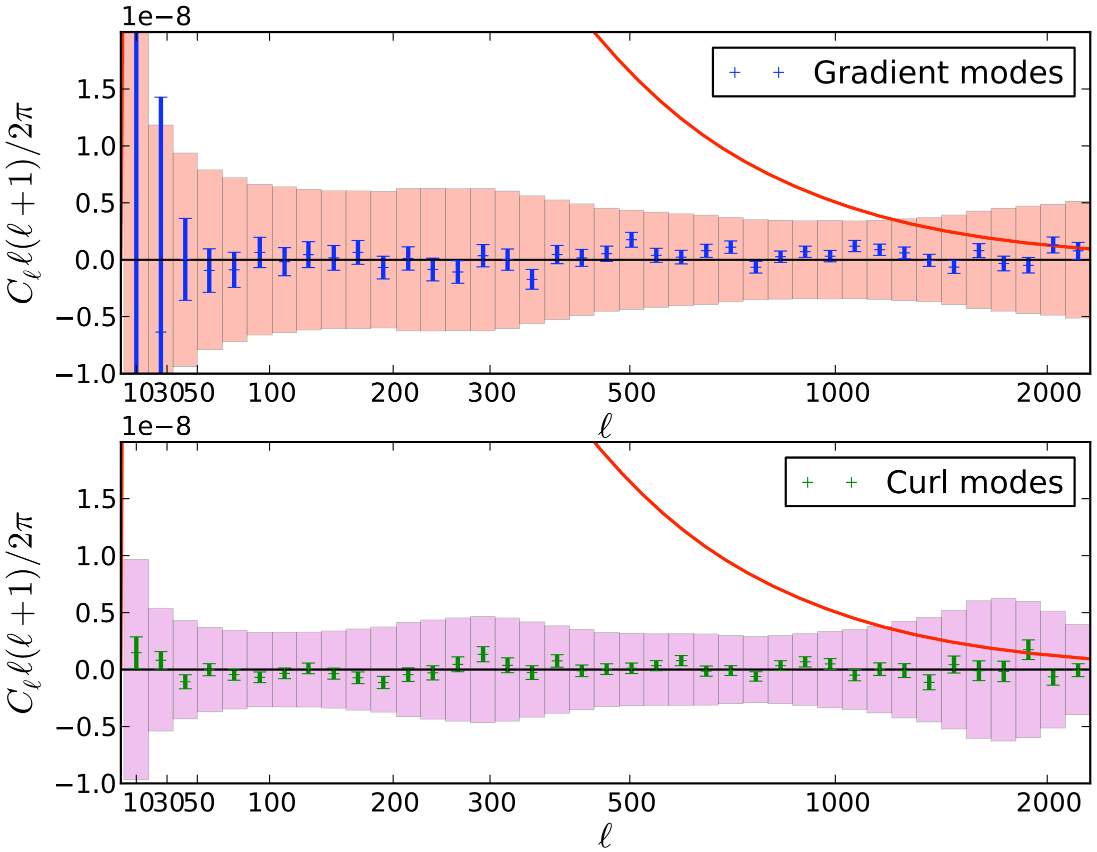
Null-test

Point sources inpainting on unlensed maps



Lensing potential reconstruction

Unlensed masked maps



Unlensed masked and inpainted maps