An algorithm for reconstructing gradient– and curl– type deflection angle from CMB maps

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Collaboration with

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Motivation of our work

➢ Deflection angle

- ✓ If we consider the lensing effect arising from the linear matter density fluctuations, the deflection angle is related to the lensing potential as

$$d_i(\vec{n}) = \partial_i \phi(\vec{n})$$

- ✓ This relation is assumed in several reconstruction methods

(e.g., Hu & Okamoto '02)

➢ Curl-type deflection angle

- ✓ In general, deflection angle has two components

2D Levi-Chivita tensor

$$d_i(\vec{n}) = \underbrace{\partial_i \phi(\vec{n})}_{\text{Gradient part}} + \underbrace{\epsilon_{ij}\partial_j \omega(\vec{n})}_{\text{Curl part}}$$

Gradient part — Curl part

- ✓ Curl-mode is non-zero if the lensing effect is induced by vector/tensor metric perturbations (e.g., cosmic string, primordial gravitational wave)

To probe physics generating the curl mode, we need a method for reconstructing curl mode from observational data

Purpose 1

Find an algorithm for reconstructing deflection angle including both gradient and curl part

- **Previous works which consider curl-type deflection angle**
  - Hirata & Seljak '03 ・Based on the likelihood estimator
  - Cooray+ '05 ・Based on the optimal quadratic estimator proposed by Hu & Okamoto '02 (HO02)
- **Our work**
  - ✓ Our estimator is based on Okamoto & Hu '03 (OH03), but including curl-type deflection angle (extension of Cooray+'05 in full sky)
  - ✓ Then, we show that the gradient- and curl-type deflection angle can be reconstructed with unbiased condition

Purpose 2

- **Sources of curl-type deflection angle**

An example: cosmic string

- ✓ **Cosmic string can be produced by the phase transition in the early universe**

- ✓ **The primordial CMB temperature anisotropies produced by cosmic strings are less than ~10% (corresponds to a constraint on dimensionless string tension: $G\mu < O(10^{-7})$)**

(e.g., Wyman+'05, Seljak+'06, Bevis+ '07)

- ✓ **Cosmic string induces vector/tensor perturbations and would produce curl-type deflection angle : cosmic string would be constrained from curl mode**

Estimate expected constraint on properties of cosmic string by reconstructing curl-type deflection angle

Brief Review of OH'03

Definition of estimator

$$\hat{\phi}_{\ell m}^{(XY)} = (-1)^m \sum_{L_1 M_1} \sum_{L_2 M_2} f_{\ell L_1 L_2}^{XY} \begin{pmatrix} \ell & L_1 & L_2 \\ -m & M_1 & M_2 \end{pmatrix} \tilde{X}_{L_1 M_1} \tilde{Y}_{L_2 M_2}$$

where $\tilde{X}_{\ell m}$ and $\tilde{Y}_{\ell m}$ is $\tilde{\Theta}_{\ell m}$, $\tilde{E}_{\ell m}$, or $\tilde{B}_{\ell m}$

To determine the functional form of f theoretically, the following conditions are imposed :

1. Unbiased

Ensemble average over the estimator $\hat{\phi}_{\ell m}^{XY}$ *with fixing the lensing potential* should be equals to the lensing potential

$$\left\langle \hat{\phi}_{\ell m}^{XY} \right\rangle_{CMB} = \phi_{\ell m}$$

2. Optimal

Choosing f so that N_ℓ is minimized

$$\left\langle \hat{\phi}_{\ell m}^{(XY)} \left(\hat{\phi}_{\ell m}^{(XY)} \right)^* \right\rangle = \underline{N_\ell^{\phi,(XY)}} + C_\ell^{\phi\phi}$$

Brief Review of OH'03

➢ Functional form of *f*

✓ **described by the observed (lensed) power spectra, $\hat{\mathcal{C}}_\ell^{XY}$, and unlensed Cl's**

$$f^{XY}_{\ell L_1 L_2} = (2\ell+1)\frac{F^{XY}_{\ell L_1 L_2}}{[\Phi F]^{XY}_\ell}$$

Summation : $\sum_{L_1}\sum_{L_2} \Phi^{XY}_{\ell L_1 L_2} F^{XY}_{\ell L_1 L_2}$

$$F^{XY}_{\ell L_1 L_2} = \frac{\hat{\mathcal{C}}^{XX}_{L_2}\hat{\mathcal{C}}^{YY}_{L_1}\Phi^{XY}_{\ell L_1 L_2} - (-1)^{\ell+L_1+L_2}\hat{\mathcal{C}}^{XY}_{L_1}\hat{\mathcal{C}}^{XY}_{L_2}\Phi^{XY}_{\ell L_2 L_1}}{\hat{\mathcal{C}}^{XX}_{L_1}\hat{\mathcal{C}}^{YY}_{L_2}\hat{\mathcal{C}}^{XX}_{L_2}\hat{\mathcal{C}}^{YY}_{L_1} - \left(\hat{\mathcal{C}}^{XY}_{L_1}\hat{\mathcal{C}}^{XY}_{L_2}\right)^2}$$

*** The quantity Φ depends on unlensed Cl's**

➢ Reconstruction

Observed anisotropies $\tilde{\Theta}_{\ell m}, \tilde{E}_{\ell m}, \tilde{B}_{\ell m}$

$$\hat{\phi}^{(XY)}_{\ell m} = (-1)^m \sum_{L_1 M_1}\sum_{L_2 M_2} f^{XY}_{\ell L_1 L_2}\begin{pmatrix} \ell & L_1 & L_2 \\ -m & M_1 & M_2 \end{pmatrix}\tilde{X}_{L_1 M_1}\tilde{Y}_{L_2 M_2}$$

✓ **In principle, we can reconstruct the lensing potential from observed CMB maps.**

Our Estimator

➢ Definition of estimators

$$\hat{\phi}^{(XY)}_{\ell m} = (-1)^m \sum_{L_1 M_1} \sum_{L_2 M_2} f^{XY}_{\ell L_1 L_2} \begin{pmatrix} \ell & L_1 & L_2 \\ -m & M_1 & M_2 \end{pmatrix} \tilde{X}_{L_1 M_1} \tilde{Y}_{L_2 M_2}$$

$$\hat{\omega}^{(XY)}_{\ell m} = (-1)^m \sum_{L_1 M_1} \sum_{L_2 M_2} g^{XY}_{\ell L_1 L_2} \begin{pmatrix} \ell & L_1 & L_2 \\ -m & M_1 & M_2 \end{pmatrix} \tilde{X}_{L_1 M_1} \tilde{Y}_{L_2 M_2}$$

where $\tilde{X}_{\ell m}$ and $\tilde{Y}_{\ell m}$ is $\tilde{\Theta}_{\ell m}, \tilde{E}_{\ell m}$, or $\tilde{B}_{\ell m}$

To determine the functional form of *f* and *g* theoretically, the following conditions are imposed :

1. Unbiased

Ensemble average over the estimators $\hat{\phi}^{XY}_{\ell m}$ and $\hat{\omega}^{XY}_{\ell m}$ *with fixing the lensing fields* should be equals to the lensing fields, respectively

$$\left\langle \hat{\phi}^{XY}_{\ell m} \right\rangle_{CMB} = \phi_{\ell m} \qquad \left\langle \hat{\omega}^{XY}_{\ell m} \right\rangle_{CMB} = \omega_{\ell m}$$

2. Optimal

Choosing f and g so that N_ℓ is minimized

$$\left\langle \hat{\phi}^{(XY)}_{\ell m} \left(\hat{\phi}^{(XY)}_{\ell m} \right)^* \right\rangle = \underline{N^{\phi,(XY)}_\ell} + C^{\phi\phi}_\ell \qquad \left\langle \hat{\omega}^{(XY)}_{\ell m} \left(\hat{\omega}^{(XY)}_{\ell m} \right)^* \right\rangle = \underline{N^{\omega,(XY)}_\ell} + C^{\omega\omega}_\ell$$

Our Estimator

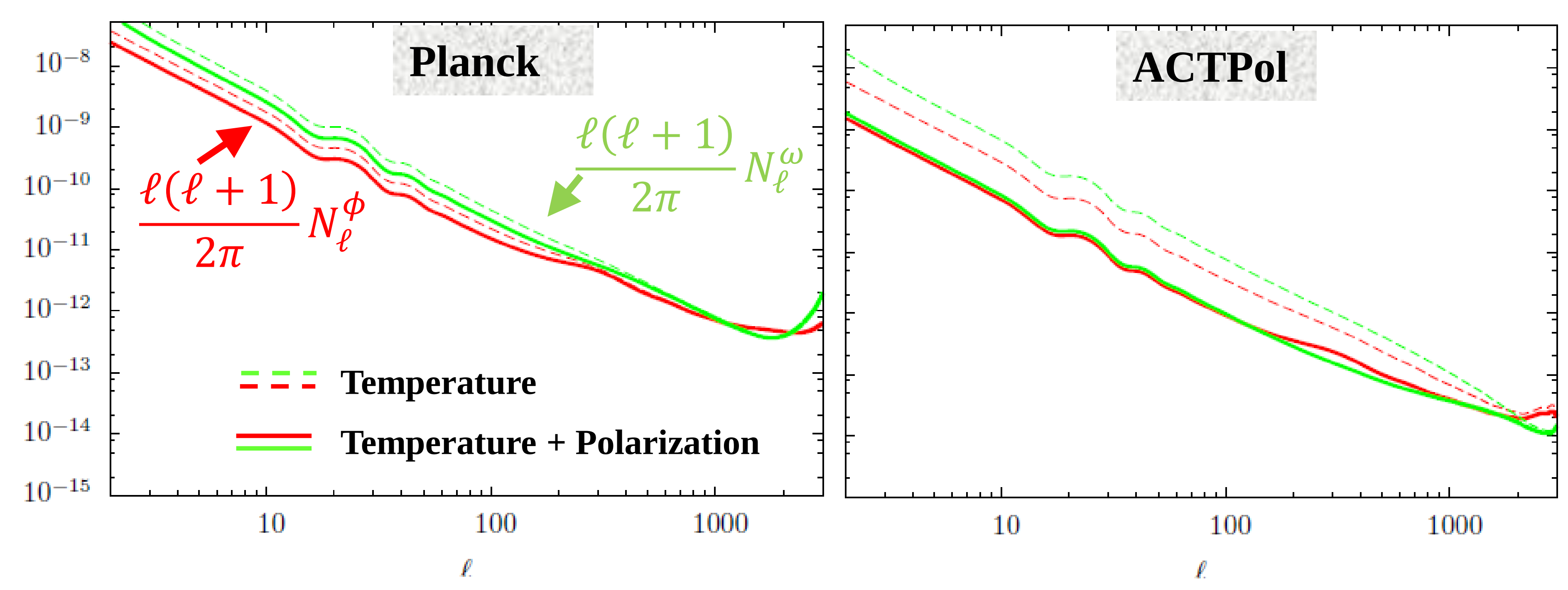
- **Functional form of f and g**
  - ✓ **Both f and g are described by the observed (lensed) and unlensed Cl's**
  - ✓ **Thanks to the property of parity, the estimators, $\hat{\phi}^{XY}_{\ell m}$ and $\hat{\omega}^{XY}_{\ell m}$ are separately described, and f is the same as that of OH'03**
  - ✓ **The functional form of g is similar to that of f**

$$f^{XY}_{\ell L_1 L_2} = (2\ell+1)\frac{F^{XY}_{\ell L_1 L_2}}{[\Phi F]^{XY}_{\ell}} \qquad F^{XY}_{\ell L_1 L_2} = \frac{\hat{C}^{XX}_{L_2}\hat{C}^{YY}_{L_1}\Phi^{XY}_{\ell L_1 L_2} - (-1)^{\ell+L_1+L_2}\hat{C}^{XY}_{L_1}\hat{C}^{XY}_{L_2}\Phi^{XY}_{\ell L_2 L_1}}{\hat{C}^{XX}_{L_1}\hat{C}^{YY}_{L_2}\hat{C}^{XX}_{L_2}\hat{C}^{YY}_{L_1} - \left(\hat{C}^{XY}_{L_1}\hat{C}^{XY}_{L_2}\right)^2}$$

$$g^{XY}_{\ell L_1 L_2} = (2\ell+1)\frac{G^{XY}_{\ell L_1 L_2}}{[\Omega G]^{XY}_{\ell}} \qquad G^{XY}_{\ell L_1 L_2} = \frac{\hat{C}^{XX}_{L_2}\hat{C}^{YY}_{L_1}\Omega^{XY}_{\ell L_1 L_2} - (-1)^{\ell+L_1+L_2}\hat{C}^{XY}_{L_1}\hat{C}^{XY}_{L_2}\Omega^{XY}_{\ell L_2 L_1}}{\hat{C}^{XX}_{L_1}\hat{C}^{YY}_{L_2}\hat{C}^{XX}_{L_2}\hat{C}^{YY}_{L_1} - \left(\hat{C}^{XY}_{L_1}\hat{C}^{XY}_{L_2}\right)^2}$$

*** The quantity Ω depends on unlensed Cl's but the dependence is different from Φ**

Noise Spectra



Note: For ACTPol, the noise improvement by including polarization is significant compared to that of Planck

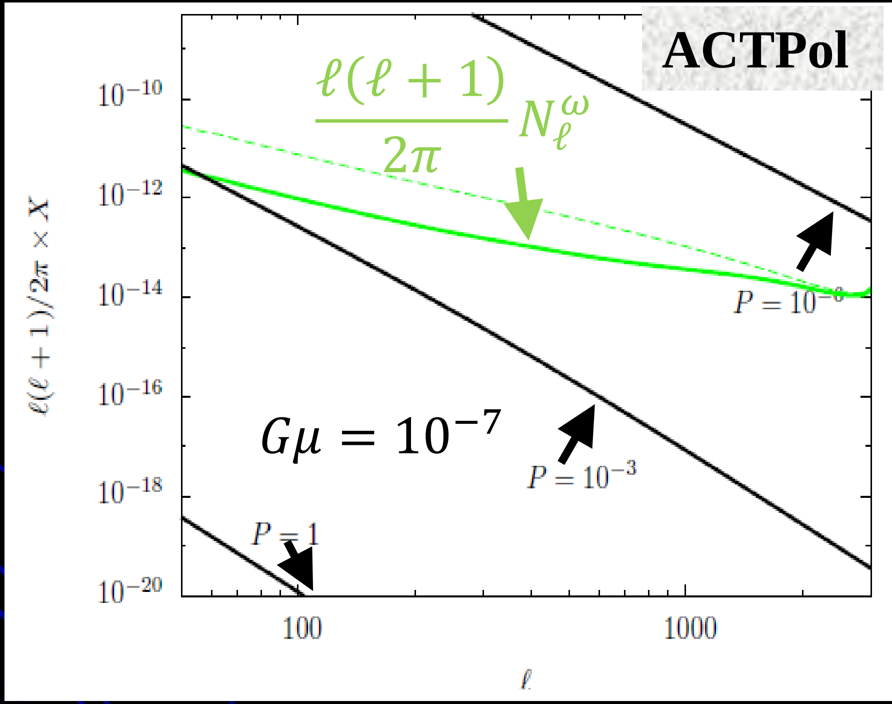
The noise of curl mode is comparable to that of gradient mode

Implications for cosmic string

➢ Assumptions

- ✓ Nambu-string
- ✓ VOS model (Martins+'02)
- ✓ Energy loss rate (Martins+'02,'04) $\sim 0.23 P v_{rms} \rho_{str}/\xi$
- ✓ Number of string in the region $[z, z+\delta z]$ is $\delta z(\frac{dV}{dz})/\xi^3$
- ✓ Straight string

➢ Results



$$\ell^2 C_\ell^{\omega\omega} \propto (G\mu)^2 P^{-\frac{5}{2}} \ell^{-5}$$

If $P < 10^{-3}$ and $G\mu > 10^{-7}$, the curl-type deflection angle induced by cosmic string would be detected

Summary

- ✓ **We show an algorithm for reconstructing deflection angle including both gradient and curl mode**

  Then, thanks to property of parity, the gradient and curl mode can be reconstructed separately.

- ✓ **Assuming ACTPol, we roughly estimate the expected constraint on cosmic string using the curl mode.**

  Using ACTPol data, if $G\mu > O(10^{-7})$ and $P < O(10^{-3})$, the curl-type deflection angle from cosmic string would be detected

  Curl mode has no contribution from liner-matter density fluctuations, so in this respect, considered as pure signal of string, which is an advantage of this method compared to other probes of string

Our algorithm provides opportunities to probe the physics which induce curl mode of deflection angle

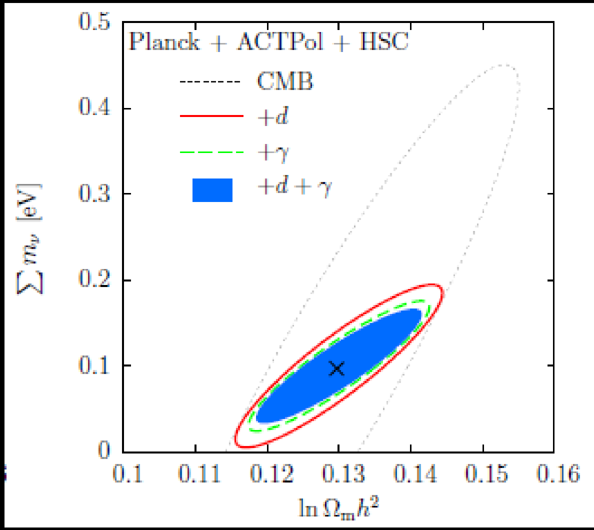
Appendix

Assumptions for cosmic string

- ✓ **Nambu-string**
- ✓ **VOS model (Martins+'02)**
- ✓ **Energy loss rate (Martins+'02,'04)** $\sim 0.23 P v_{rms} \rho_{str} / \xi$
- ✓ **Number of string in the region $[z, z + \delta z]$ is $\delta z (\frac{dV}{dz}) / \xi^3$**
- ✓ **Straight string**

What can we probe with CMB lensing ?

- **Weak lensing as a probe of dark energy, massive neutrinos, …**
  - ✓ Sensitive to high-z structure
  - ✓ Properties of source (CMB) are well known
  - ✓ Complementary to other probes



TN, Saito, Taruya '10

- **Primordial gravitational wave**
  - ✓ On small scales, B-mode is dominated by lensing.
  - ✓ Constraints on r would be improved by extracting lensing B-mode.

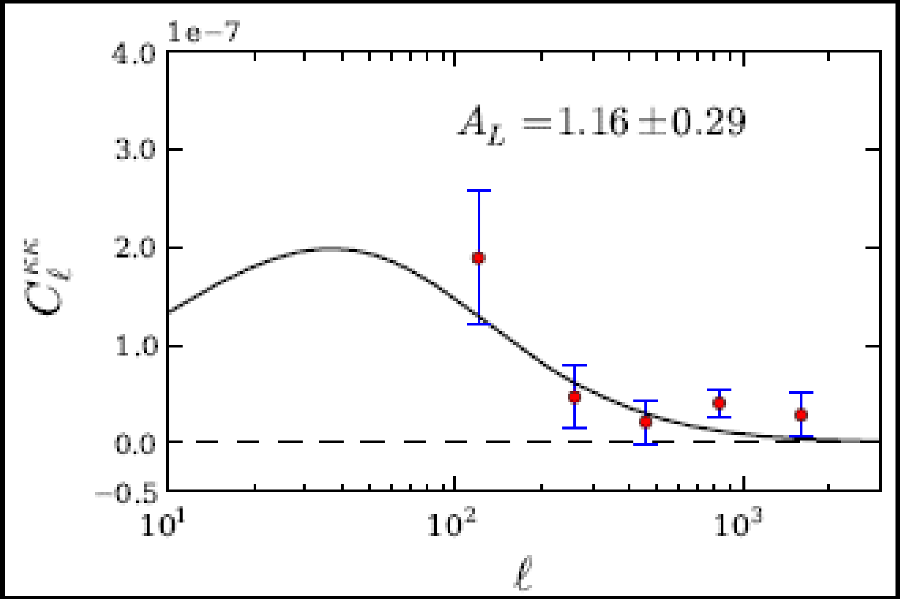
- **Some sources which induce Curl-type deflection angle**
  - ✓ Cosmic string, gravitational wave, …

Measurement of CMB lensing

- **detection**

$C_\ell^{g\kappa}$
- Smith+ '07 (3.4σ)
- Hirata+ '08 (2.5σ)

$C_\ell^{\kappa\kappa}$
- Smidt+ '10 (~2σ)
- Das+ '11 (~4σ)



Das+ '11

- **Upcoming, future experiments**
  - ✓ Ground
    - PolarBear (2011-)
    - ACTPol (2012-)
  - ✓ Space
    - Planck (2010-)
    - CMBPol (?)

CMB lensing would be detected high accuracy enough to provide us cosmological implications

Note: In Cooray +'05, they claim their estimator is not satisfied this condition, but I checked their estimator satisfies the condition.

$$\Phi^{\Theta\Theta}_{\ell L_1 L_2} = {}_0\mathcal{F}^{\phi}_{\ell L_2 L_1} C^{\Theta\Theta}_{L_2} + {}_0\mathcal{F}^{\phi}_{\ell L_1 L_2} C^{\Theta\Theta}_{L_1} \qquad \Omega^{\Theta\Theta}_{\ell L_1 L_2} = {}_0\mathcal{F}^{\omega}_{\ell L_2 L_1} C^{\Theta\Theta}_{L_2} - {}_0\mathcal{F}^{\omega}_{\ell L_1 L_2} C^{\Theta\Theta}_{L_1}$$

$$s\mathcal{F}^{\phi}_{\ell L_1 L_2} = \sqrt{\frac{(2\ell+1)(2L_1+1)(2L_2+1)}{16\pi}}[-L_2(L_2+1)+\ell(\ell+1)+L_1(L_1+1)] \times \begin{pmatrix} L_2 & \ell & L_1 \\ s & 0 & -s \end{pmatrix}$$

$$s\mathcal{F}^{\phi}_{\ell L_1 L_2} = -i\sqrt{\frac{(2\ell+1)(2L_1+1)(2L_2+1)}{16\pi}}\sqrt{\ell(\ell+1)} \times \left[\sqrt{(L_1+s)(L_1-s+1)}\begin{pmatrix} L_2 & \ell & L_1 \\ s & -1 & -s+1 \end{pmatrix} - \sqrt{(L_2-s)(L_2+s+1)}\begin{pmatrix} L_2 & \ell & L_1 \\ s & 1 & -s-1 \end{pmatrix}\right]$$

A hint of reconstruction of the curl part

- **Average only "primary CMB" anisotropies**

Similar analogy to HO'02

$$\langle\widetilde{\Theta}_{L_1M_1}\widetilde{\Theta}_{L_2M_2}\rangle_{CMB} = C^{\Theta\Theta}_{L_1}\delta_{L_1L_2}\delta_{M_1M_2}(-1)^{M_1} + \sum_{\ell m}(-1)^m[\Phi^{\Theta\Theta}_{\ell L_1 L_2}\phi_{\ell m} + \Omega^{\Theta\Theta}_{\ell L_1 L_2}\omega_{\ell m}]\begin{pmatrix}\ell & L_1 & L_2\\ -m & M_1 & M_2\end{pmatrix}$$

[Key property]

$$\ell + L_1 + L_2 = \text{odd}\,,\quad \Phi_{\ell L_1 L_2} = 0$$

$$\ell + L_1 + L_2 = \text{even}\,,\quad \Omega_{\ell L_1 L_2} = 0$$

→ **$\phi_{\ell m}$, $\omega_{\ell m}$ are expressed independently**

For ω,

$$\omega_{\ell m} = (2\ell+1)(-1)^m \times \sum_{L_1M_1}\sum_{L_2M_2}\frac{G^{\Theta\Theta}_{\ell L_1L_2}}{[\Omega G]^{\Theta\Theta}_{\ell}}\begin{pmatrix}\ell & L_1 & L_2\\ -m & M_1 & M_2\end{pmatrix}\langle\widetilde{\Theta}_{L_1M_1}\widetilde{\Theta}_{L_2M_2}\rangle_{CMB}$$

Arbitrary function (→ $G^{\Theta\Theta}_{\ell L_1L_2}$)

Estimator including curl-mode

- **definition**

$$\phi_{\ell m}^{(XY)} = (-1)^m \sum_{L_1 M_1} \sum_{L_2 M_2} f_{\ell L_1 L_2}^{XY} \begin{pmatrix} \ell & L_1 & L_2 \\ -m & M_1 & M_2 \end{pmatrix} \tilde{X}_{L_1 M_1} \tilde{Y}_{L_2 M_2}$$

$$\omega_{\ell m}^{(XY)} = (-1)^m \sum_{L_1 M_1} \sum_{L_2 M_2} g_{\ell L_1 L_2}^{XY} \begin{pmatrix} \ell & L_1 & L_2 \\ -m & M_1 & M_2 \end{pmatrix} \tilde{X}_{L_1 M_1} \tilde{Y}_{L_2 M_2}$$

To determine *f* and *g*, we impose the following conditions

1. Unbiased estimator

$$\langle \phi_{\ell m}^{XY} \rangle = \phi_{\ell m}$$

$$\langle \omega_{\ell m}^{XY} \rangle = \omega_{\ell m}$$

Note: In Cooray +'05, they claim their estimator is not satisfied this condition, but I checked their estimator satisfies the condition.

2. Optimal estimator

Note: we only consider 1st order of Taylor expansion of anisotropies with respect to lensing fields

Choosing f and g so that N_ℓ is minimized

$$\left\langle \phi_{\ell m}^{(XY)} \left(\phi_{\ell m}^{(XY)}\right)^* \right\rangle = N_\ell^{\phi,(XY)} + C_\ell^{\phi\phi} \qquad \left\langle \omega_{\ell m}^{(XY)} \left(\omega_{\ell m}^{(XY)}\right)^* \right\rangle = N_\ell^{\omega,(XY)} + C_\ell^{\omega\omega}$$

Our Estimator

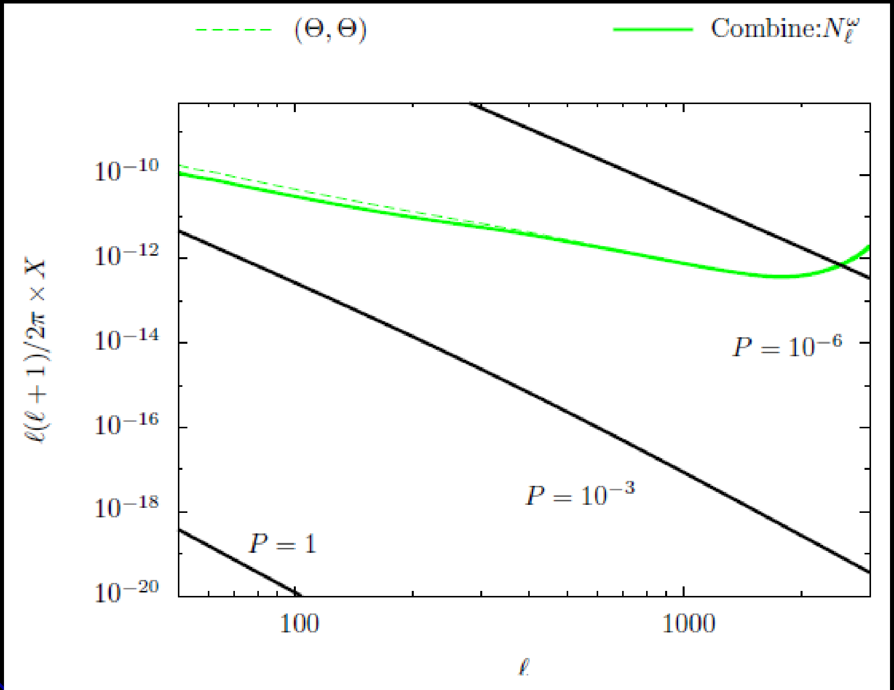
- **Functional form of f and g**

Observed anisotropies $\tilde{\Theta}_{\ell m}, \tilde{E}_{\ell m}, \tilde{B}_{\ell m}$

$$G^{XY}_{\ell L_1 L_2} = \frac{\hat{C}^{XX}_{L_2}\hat{C}^{YY}_{L_1}\Omega^{XY}_{\ell L_1 L_2} - (-1)^{\ell+L_1+L_2}\hat{C}^{XY}_{L_1}\hat{C}^{XY}_{L_2}\Omega^{XY}_{\ell L_2 L_1}}{\hat{C}^{XX}_{L_1}\hat{C}^{YY}_{L_2}\hat{C}^{XX}_{L_2}\hat{C}^{YY}_{L_1} - \left(\hat{C}^{XY}_{L_1}\hat{C}^{XY}_{L_2}\right)^2}$$

$$g^{XY}_{\ell L_1 L_2} = (2\ell+1)\frac{G^{XY}_{\ell L_1 L_2}}{[\Omega G]^{XY}_{\ell}}$$

$$\omega^{(XY)}_{\ell m} = (-1)^m \sum_{L_1 M_1}\sum_{L_2 M_2} g^{XY}_{\ell L_1 L_2}\begin{pmatrix} \ell & L_1 & L_2 \\ -m & M_1 & M_2 \end{pmatrix}\tilde{X}_{L_1 M_1}\tilde{Y}_{L_2 M_2}$$



(Θ, Θ)
Combine:N_ℓ^ω
10^{-10}
10^{-12}
10^{-14}
10^{-16}
10^{-18}
10^{-20}
$\ell(\ell+1)/2\pi \times X$
$P = 10^{-6}$
$P = 10^{-3}$
$P = 1$
100
1000
ℓ