The Shape of the CMB Lensing Bispectrum

Duncan Hanson

Berkeley Lensing Workshop, April 22, 2011

"The shape of the CMB lensing bispectrum"
Lewis, Challinor, Hanson (2011) arXiv:1101.2234

"CMB lensing and primordial non-Gaussianity"
Hanson, Smith, Challinor, Liguori (2009) arXiv:0905.4732

OUTLINE

I INTRODUCTION

- Lensing Bispectra
- Overlap with $f_{NL}^{\rm local}$

II BISPECTRUM GUIDE:

- Significance in the high S/N limit
- Effect of lensing on the shape of other bispectra (particularly $f_{NL}^{\rm local}$)

III PROSPECTS:

- Current status, ultimate limits.
- Planck

THE T-ϕ CORRELATION

The CMB temperature is given by

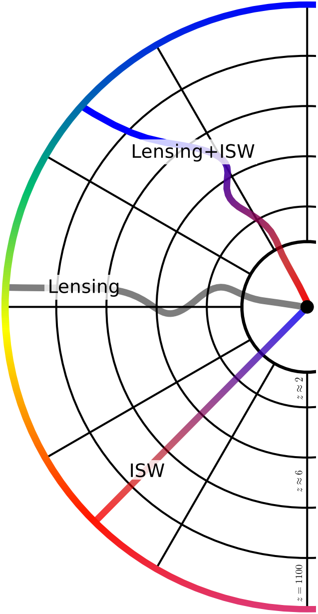
$$T(\hat{n}) = \Theta(\hat{n}) + \Theta^{ISW}(\hat{n}) + [\nabla\phi \cdot \nabla\Theta](\hat{n}) + \ldots,$$

where the ISW and lensing effects are

$$\begin{aligned} \Theta^{ISW}(\hat{n}) &= 2\int_0^{\chi_*} d\chi\, \dot{\Psi}(\chi\hat{n}; \eta_0 - \chi) \\ \phi(\hat{n}) &= -2\int_0^{\chi_*} d\chi \frac{\chi_* - \chi}{\chi_* \chi} \Psi(\chi\hat{n}; \eta_0 - \chi) \end{aligned}$$

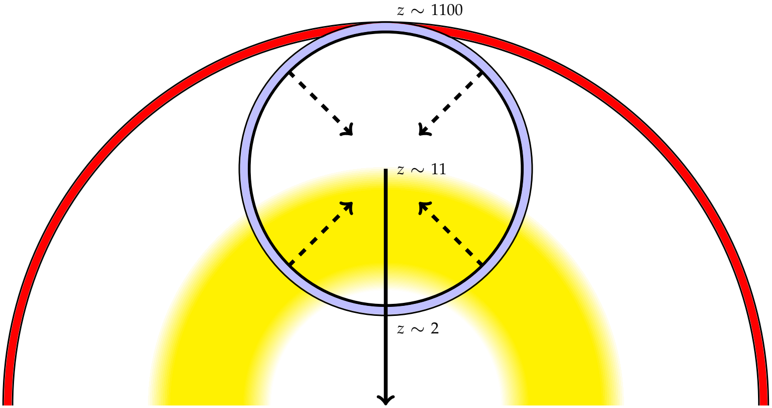
This leads to a non-Gaussian bispectrum:

$$\langle TTT \rangle = \left\langle \Theta^{ISW} (\nabla\phi \cdot \nabla\Theta)\Theta \right\rangle$$



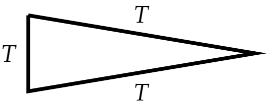
THE E-ϕ CORRELATION

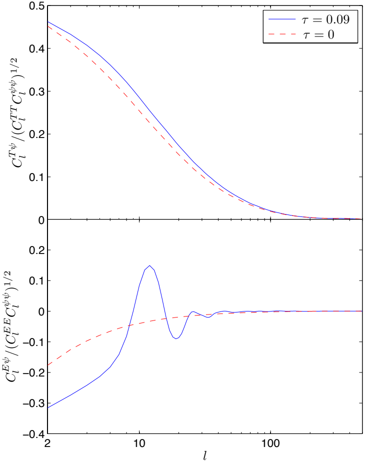
There are also lensing bispectra in polarization. No longer from cross-correlation with ISW, but from overlap between potentials which source ϕ and quadrupoles which source reionization E-modes (Lewis, Challinor, Hanson 2011). Recently implemented in CAMB.



THE LENSING BISPECTRUM

- The T-ϕ and E-ϕ correlations are significant– $\mathcal{O}(30\%)$ on large scales, but fade quickly.
- The ϕ-induced T and E covariances are mostly at high-l, while the cross-correlation is at low-l, so get a squeezed shape.

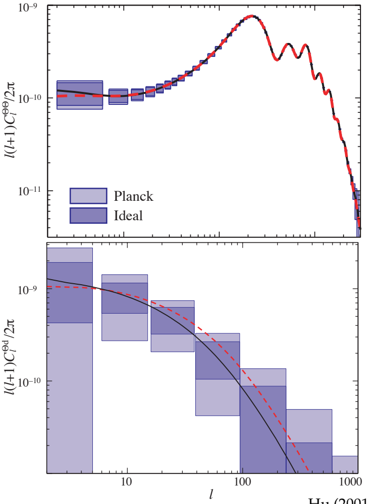




Lewis, Challinor, Hanson (2011)

USE OF LENSING-ISW

- The lensing-ISW bispectrum is a (relatively) direct probe of dark energy (Seljak and Zaldarriaga 1998, Goldberg and Spergel 1998).
- Particularly good at breaking the angular diamaeter distance degeneracy (Hu 2001).
- There is a significant overlap with the $f_{NL}^{\rm local}$-type bispectrum $\Psi^{NG}(\vec{x}) = \Psi + f_{NL}(\Psi^2 - \langle\Psi^2\rangle)$ (Smith and Zaldarriaga 2006).

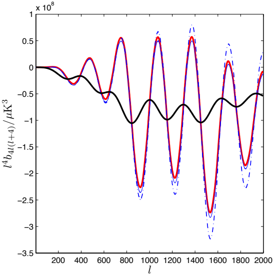


Hu (2001)

Lensing-ISW and f_{NL}^{local}

Why does the lensing-ISW bispectrum project onto the f_{NL}^{local} bispectrum?

- Lensing convergence results in a local change of scale → local change of variance.
- f_{NL}^{local} corresponds to a local change in the amplitude of the power spectrum → local change of variance.



So there is an overlap between the local and ISW bispectra (although phases differ). The large amplitude of the ISW-lensing bispectrum results in significant contamination for f_{NL}^{local}.

THINGS TO WORRY ABOUT

Planck has the ability to detect this signal (plots to come later), but there are a few fundamental things to worry about:

- In the event of a significant detection, what is the effect of signal variance?
- What is the effect of lensing on other bispectra (claims of large effects from Cooray, Sarkar and Serra 2008), and is there a way to calculate the lensing of the bispectrum non-perturbatively?

  Higher order lensing terms in $C_l^{\hat{\phi}\hat{\phi}}$ are known to be important for Planck, SPT (e.g. Alex's talk yesterday).

Birdwatchers' Guide to Bispectrum Calculations

We'll write the lensed temperature graphically:

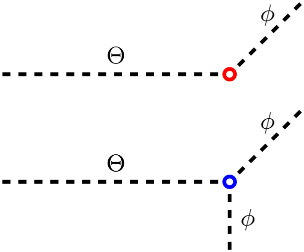
$$T(\hat{n}) = \Theta(\hat{n}) + \Theta^{ISW}(\hat{n}) + \nabla^i\phi\nabla_i\Theta(\hat{n}) + \nabla^i\phi\nabla^j\phi\cdot\nabla_{ij}\Theta(\hat{n}) + \dots$$

Using the following glossary:

Observable field:
T

Gaussian field:
Θ, Θ^{ISW}

Product of fields:



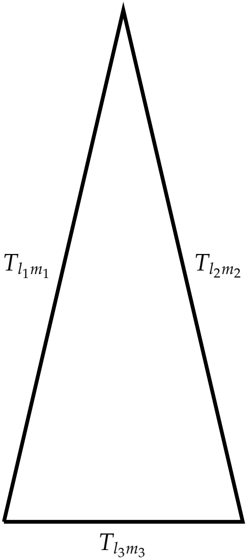
SIGNAL VARIANCE

In bispectrum-language, often think of the minimum-variance estimator for the lensing-ISW amplitude as

$$\hat{A} = \frac{1}{\mathcal{F}} \sum_{l_1 l_2 l_3} \sum_{m_1 m_2 m_3} \times \frac{T_{l_1 m_1} T_{l_2 m_2} T_{l_3 m_3}}{C_{l_1}^{TT} C_{l_2}^{TT} C_{l_3}^{TT}} B_{l_1 l_2 l_3}^{m_1 m_2 m_3}.$$

It's more natural to write

$$\hat{A} = \frac{1}{\mathcal{F}} \sum_{lm} \frac{C_l^{T\phi} \hat{\phi}_{lm} T_{lm}^*}{N_l^{\phi\phi} C_l^{TT}}.$$



Signal Variance

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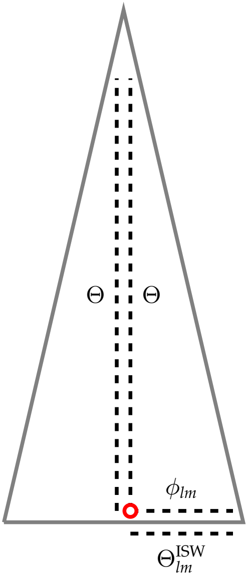


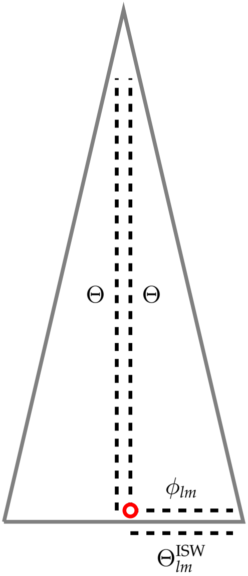
Signal Variance

Treating $\hat{\phi}$ as Gaussian, suggests the prescription

$$\hat{A} = \frac{1}{\mathcal{F}} \sum_{lm} \frac{C_l^{T\phi} \hat{\phi}_{lm} T^*_{lm}}{N_l^{\phi\phi} C_l^{TT}} \rightarrow$$

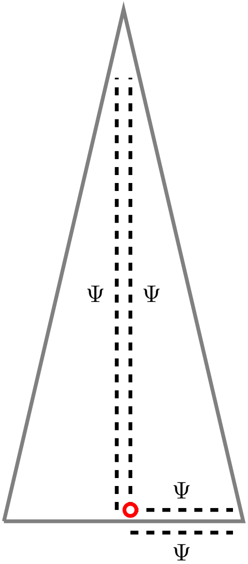
$$\hat{A} = \frac{1}{\mathcal{F}'} \sum_{lm} \frac{C_l^{T\phi} \hat{\phi}_{lm} T^*_{lm}}{(N_l^{\phi\phi} + C_l^{\phi\phi}) C_l^{TT} + (C_l^{T\phi})^2}.$$

From simulations, describes well the increase in variance due to signal.



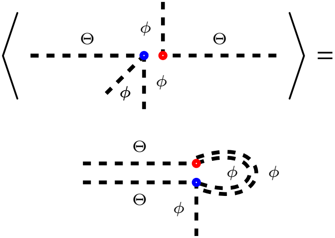
SIGNAL VARIANCE FOR f_{NL}^{local}

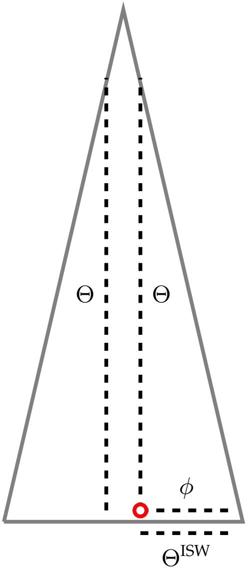
- The same picture applies to the squeezed triangles of f_{NL}^{local} as well (sort of like Munshi and Heavens 2009).
- Can picture as correlation of two small-scale modes, modulated by large-scale mode.
- Potential improvement on the CSZ approach (Creminelli et. al. 2007, Smith et. al. 2011).



Bispectrum Lensing

- For a squeezed bispectrum, safe to use the “unlensed short-leg” approximation.
- Lensing terms generate coupling between the two long-wavelength modes, e.g.





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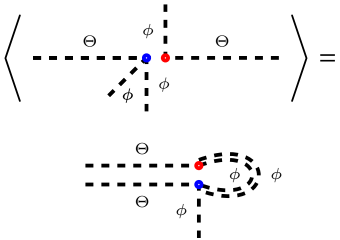
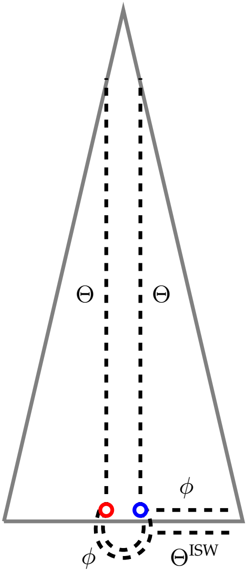
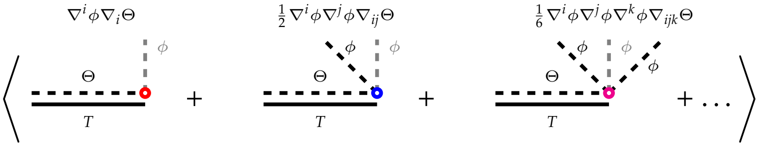




Bispectrum Lensing

We want to calculate



Always keeping one ϕ free (grayed out) to correlate with ISW.

Note that in real space

$$\frac{\delta}{\delta\nabla\phi(\hat{n})}T(\hat{n}) = (\nabla\Theta)[\hat{n} + \nabla\phi(\hat{n})] = \widetilde{\nabla T}(\hat{n}),$$

where $\widetilde{\nabla T}$ is the lensed gradient of the CMB.

Bispectrum Lensing

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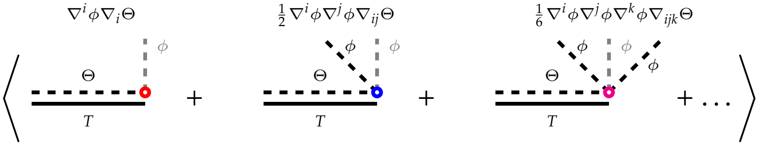


Always keeping one ϕ free (grayed out) to correlate with ISW.

In Fourier space we have (again with $\widetilde{\nabla T}(\vec{\hat{n}}) = \nabla\Theta[\hat{n} + \nabla\phi(\hat{n})]$)

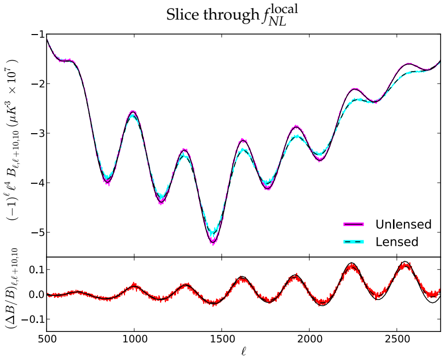
$$\left\langle T(\vec{l}_1)\frac{\delta}{\delta\phi(\vec{l}_2)^*}T(\vec{l}_3)\right\rangle = -\frac{i}{2\pi}\delta(\vec{l}_1+\vec{l}_2+\vec{l}_3)\vec{l}_2\cdot\left\langle T(\vec{l}_1)\widetilde{\nabla T}(\vec{l}_1)^*\right\rangle$$

$$\approx -\frac{1}{2\pi}\delta(\vec{l}_1+\vec{l}_2+\vec{l}_3)(\vec{l}_1\cdot\vec{l}_2)C^{TT}_{l_1}.$$

Where the validity of $\approx$ is determined by the extent to which gradients and lensing commute.

BISPECTRUM LENSING

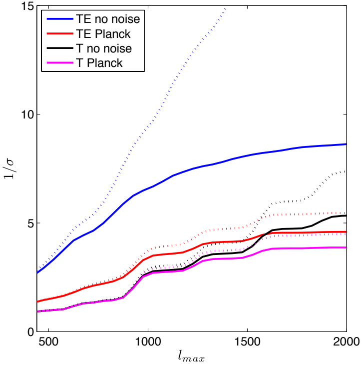
- So lensing of squeezed bispectra may be approximated simply by accurately lensing the long legs, e.g. using CAMB.
- Works well for f_{NL}^{local}.
- Corrects an $\mathcal{O}(10\%)$ suppression at low-l.
- Also explains the $N_l^{(2)}$ bias for $\hat{\phi}_{lm}$ power spectrum at low-l.



Hanson et. al. (2009)

Detection on Data

- Currently null results on WMAP (e.g. Calabrese et. al. 2009).
- Ultimate detection significance with T and E limited by signal variance to $\approx 9\sigma$, although non-linear effects could make this higher (Mangilli and Verde 2009).

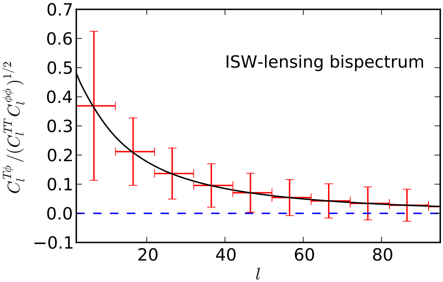


	$\sigma_{f_{\rm NL}}$	$\sigma_{\rm lens}$	Corr.	$f_{\rm NL}$ Bias	$\sigma_{f_{\rm NL}}^{\rm marge}$
T	4.31	0.19	0.24	9.5	4.44
T+E	2.14	0.12	0.022	2.6	2.14
Planck T	5.92	0.26	0.22	6.4	6.06
Planck T+E	5.19	0.22	0.13	4.3	5.23

Lewis, Challinor, Hanson (2011)

Planck Prospects

- Good match for Planck, with full-sky coverage.
- Predict $\approx 4\sigma$ from temperature alone for linear ISW.
- Difficulty in that low-l in ϕ is also where many scan-strategy associated systematics live.
- Also need to worry about things like peculiar velocity dipole.



(50μKarcmin, 7′ FWHM)

A challenge, but exciting!

Conclusions

- ISW-ϕ directly traces dark energy / matter at $z \sim 2$.
- E-ϕ is significant as well, and should be sensitive to even higher redshifts.
- Signal variance can be treated by approximating $\hat{\phi}$ as Gaussian.
- "non-Perturbative" lensed bispectra can be calculated in the squeezed limit, using unlensed short-legs.
- Aim for detection with Planck, which has sky-coverage and S/N for $\approx 4\sigma$.