High precision pixel-based simulations of CMB lensing

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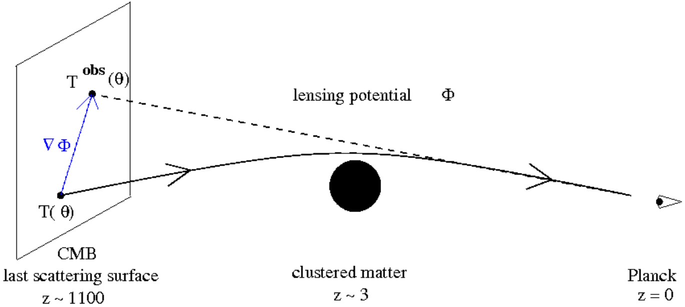
Outline

How to simulate CMB lensing

- An observed CMB field today is the primordial signal coming from another direction

$$\tilde{X}(\hat{\mathbf{n}}) = X(\hat{\mathbf{n}}') \qquad \hat{\mathbf{n}}' = \hat{\mathbf{n}} + \nabla\psi$$



- Series expansion is not accurate and slowly converging (Lewis 2005)
- Pixel based method: remapping points as function of position
- Displacement computed in Born approximation
- High resolution required!

How to simulate CMB lensing? II

1. CMB: $C_\ell^T\, C_\ell^E, C_\ell^B \rightarrow a_{lm}^X$
2. $C_\ell^\psi \rightarrow \psi_{\ell m} \rightarrow \vec{d}$

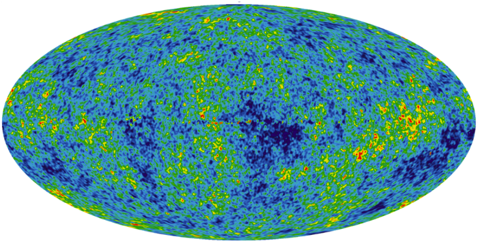
   $${}_1E_{lm}^d = \sqrt{l(l+1)}\psi_{lm} \quad {}_1B_{lm}^d = 0$$

3. $\hat{\mathbf{n}} = (\vartheta, \varphi) \rightarrow \hat{\mathbf{n}}' = (\vartheta', \phi + \Delta\phi)$

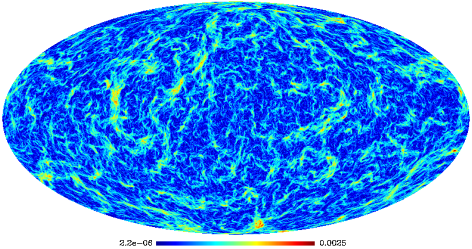
   $$\cos\vartheta' = \cos d \cos\vartheta - \sin d \sin\vartheta \cos\alpha$$

   $$\sin\Delta\varphi = \frac{\sin\alpha \sin d}{\sin\vartheta'}$$

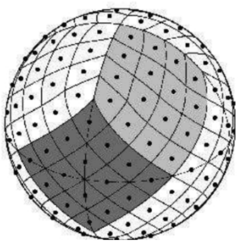
4. Resampling T, P fields at displaced position
5. Smoothing (and repixelization) according to experiment requirements



Realization of the displacement field

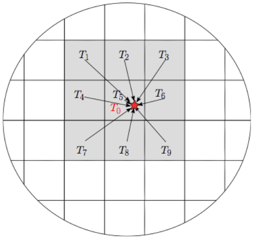


Problems and proposed solutions



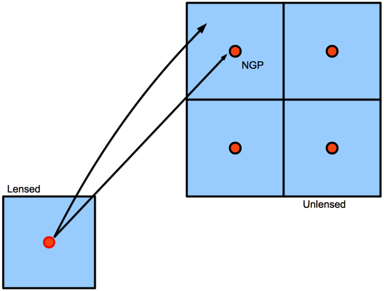
- Direct resummation
- Bicubic interpolation (LENSPix)
- Recasting on 2-D torus + NFFT (Basak et al. 2009)
- Statistical interpolation using spectral information (Lavaux et al. 2010)

- Step 1-3 can be performed with fast SHT
- Step 4 is the problem: we have to be clever
- Lensing changes band width properties
- Does grid properly sample lensed sky?



An alternative approach

- Is interpolation completely safe?
- How to keep workload constant?
- Alternative approach: Nearest Grid Point
- **Pros**: Fast, as cheap as FSHT
- **Cons**: Calls for very dense sampling



- Parallel HPSC are quickly ubiquitous
- Massively parallel solutions are more and more affordable
- We need a superior SHT algorithm

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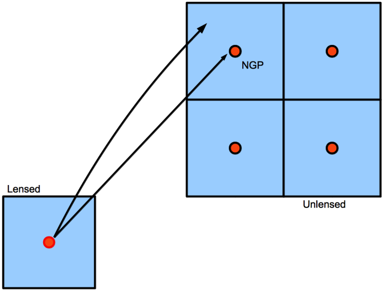

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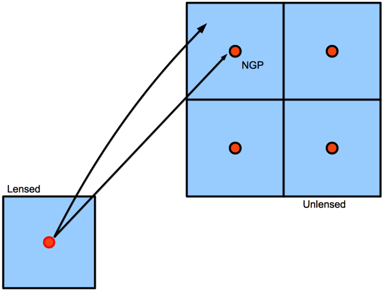

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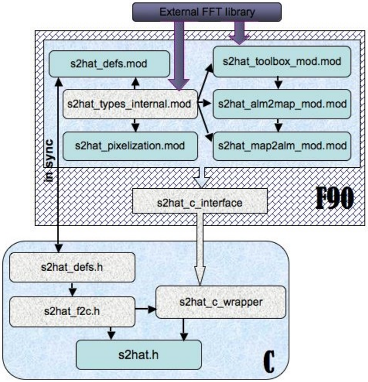
LenS^{2}HAT : Lensing with S^{2}HAT

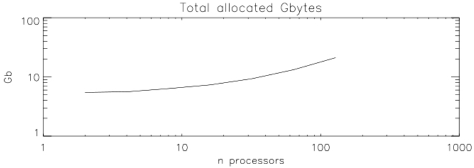
- Distributing both pixel and harmonic domain objects
- S^{2}HAT library: Fourier analysis for spin fields on the sphere

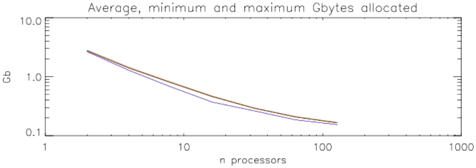


- Designed for parallel architectures
- SPRNG random numbers
- MPI Standard
- Arbitrary pixelization scheme
- Memory requirements $\mathcal{O}(10N_{pix}/n_{proc})$
- Optimized remapping
- T, TQU, QU simulations
- Sub dominant remapping cost
- Nearly perfect scaling

LensPix Vs LenS^{2}HAT

- LenS^{2}HAT memory requirements ($nside = 2048, \ell_{max} = 2048$)





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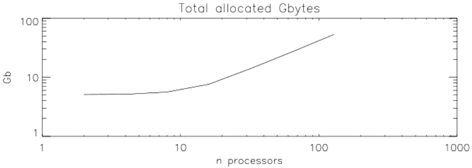

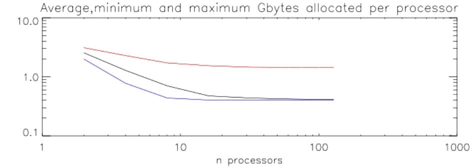

Trade-off: kernels

- Do we correctly compute lensing in the pixel domain?
- The B mode case
- $C_{\ell_1}^{BB_{lens}} = \frac{1}{2\ell_1+1} \sum_{\ell_2 \ell_3}^{\ell_{max}} |f_{\ell_1 \ell_2 \ell_3}^{EB}|^2 C_{\ell_2}^{EE} C_{\ell_3}^{\phi\phi}$
- $f_{\ell_1 \ell_2 \ell_3}^{EB} = \frac{F_{\ell_1 \ell_2 \ell}^{-2} - F_{\ell_1 \ell_2 \ell_3}^{2}}{2i}$
- $F_{\ell_1 \ell_2 \ell_3}^{s} = [-\ell_1(\ell_1+1)+\ell_2(\ell_2+1)+\ell_3(\ell_3+1)] \sqrt{\frac{(2\ell_1+1)(2\ell_2+1)(2\ell_3+1)}{16\pi}} \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ -s & s & 0 \end{pmatrix}$
- $C_{\ell_2}^{EE} = \delta_{\ell \ell_2}$
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- How LenS^{2}HAT compare?

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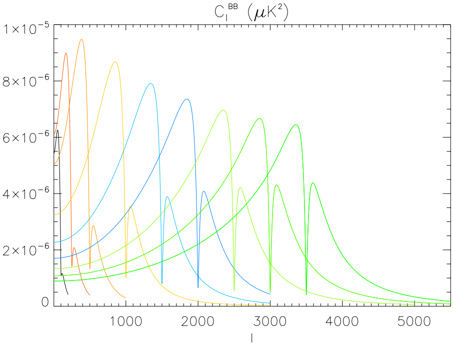

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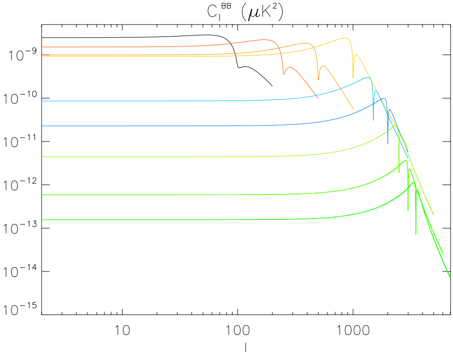

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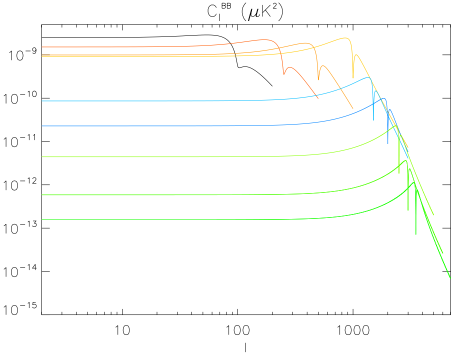
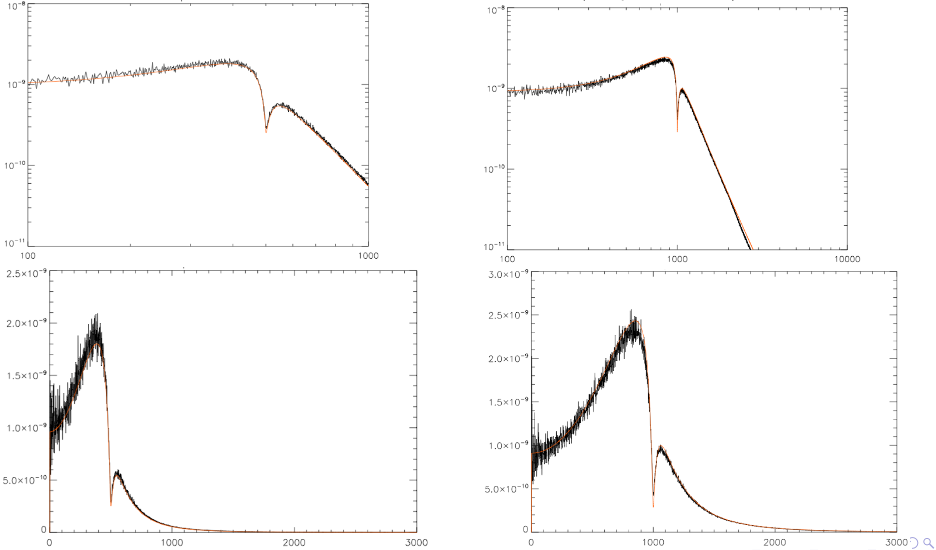
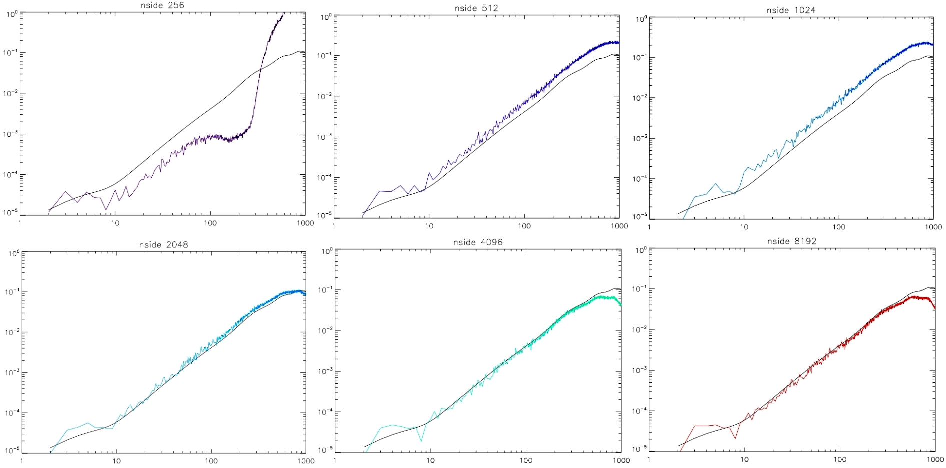


LenS^{2}HAT kernels

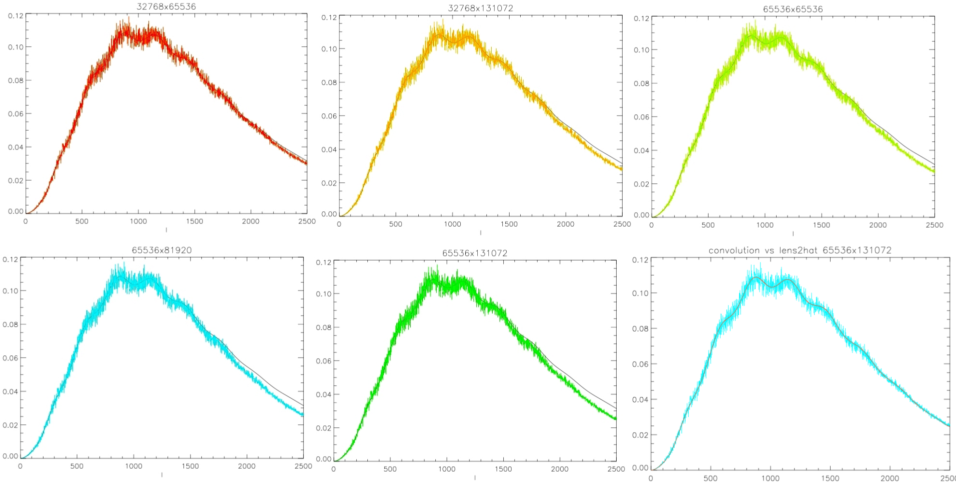
$\ell = 512, \ell_{max} = 3000, nside = 8192$ $\ell = 1000, \ell_{max} = 3000, nside = 16384$



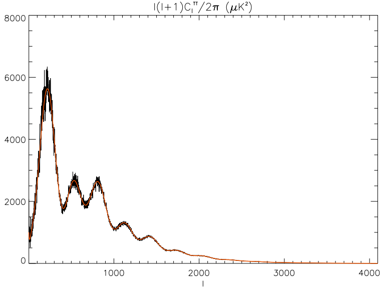
1-D convergence: HEALPix grid $\ell_{max} = 1000$

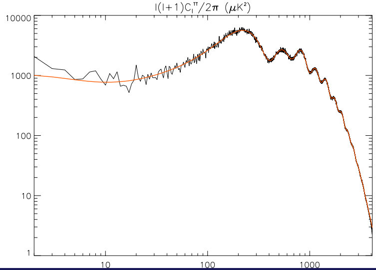


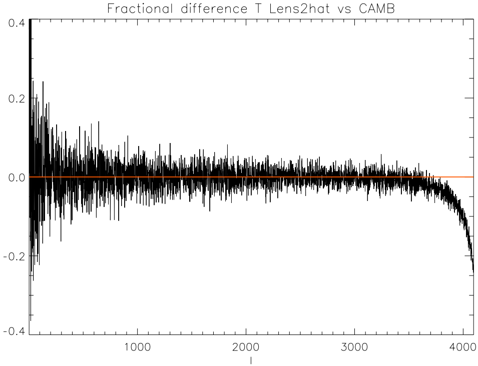
1-D convergence II: ECP grid $\ell_{max} = 2500$



Full sky LenS^{2}HAT simulations







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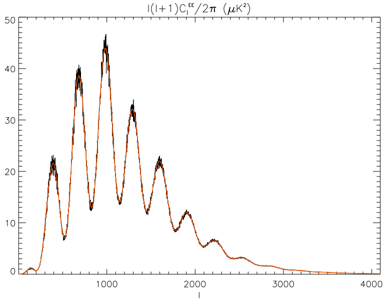

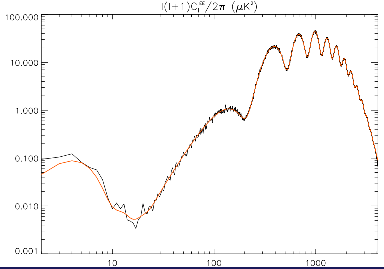

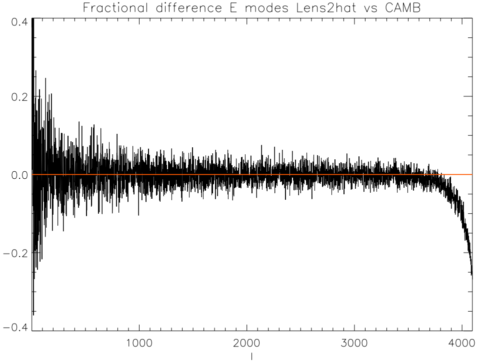

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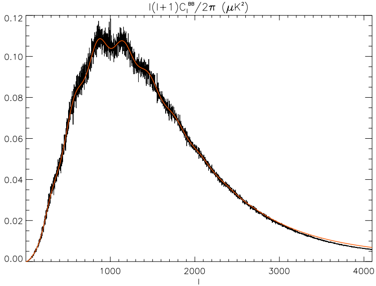
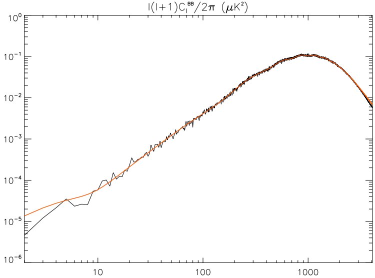
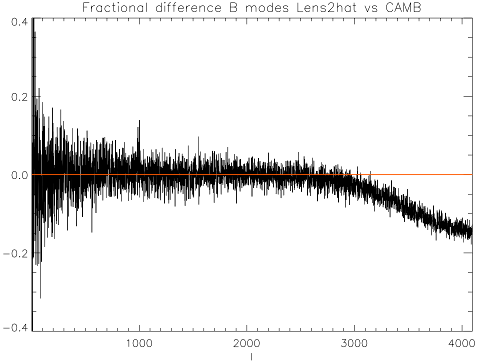
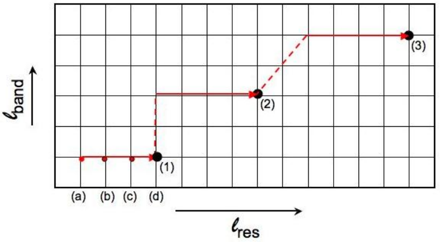






Conclusions

- NGP method
- High performance code LenS^{2}HAT
- Grid resolution and ℓ_{max} affect the B modes accuracy
- Validation of the method
- Useful for MC



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- Perfect quadrature with ECP
- SHT is the most expensive part of the method
- Towards exact method: lensing with GPUs?

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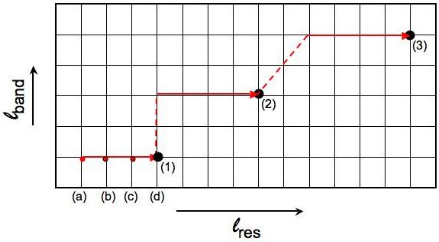

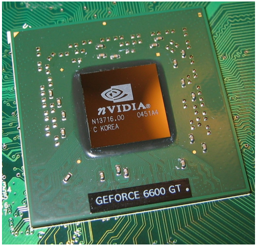

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