Reconstruction of Gravitational Lensing

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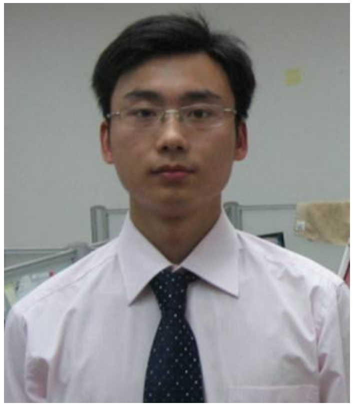


Outline

Introduction

- **Lensing introduces non-Gaussianity**

- **Non-Gaussianity permits Delensing**

- **Delensing largely removes non-Gaussianity and thus reduces**
- **bias in Cosmological parameters**

- **Delensing needed to identify primordial B mode**

Two methods

Truncated Taylor series expansion (Hirata, Hu and Okamoto)

Maximization of Likelihood (tryout)

Flat sky reconstruction

Hu & Okamoto. Astrophys. J. 574:566-574, 2002

$$X(\mathbf{n}) = \tilde{X}(\mathbf{n}+\mathbf{d}) = \tilde{X}(\mathbf{n}) + \mathbf{d}\cdot\nabla\tilde{X}(\mathbf{n}) + O(d^2)$$

$$\delta B(\mathbf{l}) = \int \frac{d^2 l'}{(2\pi)^2}[\tilde{B}(\mathbf{l}')\cos 2\phi_{\mathbf{l}'\mathbf{l}} + \tilde{E}(\mathbf{l})\sin 2\phi_{\mathbf{l}'\mathbf{l}}]W(\mathbf{l}',\mathbf{K})$$

$$W(\mathbf{l},\mathbf{K}) = -\mathbf{l}\cdot\mathbf{K}\phi(\mathbf{K})$$

K = l - l’

$$< X(\mathbf{l})X'(\mathbf{l}') >= (2\pi)^2\tilde{C}_l^{XX'}\delta(\mathbf{L}) + f_\alpha(\mathbf{l},\mathbf{l}')\phi(\mathbf{L})$$

L = l + l’ α **= TT, TE, TB, …**

f_α **is known**

Off-diagonal elements and non-Gaussianity

Hu & Okamoto's estimator for deflection angle field

$$d(\mathbf{L}) = \frac{A(L)}{L}\int \frac{d^2 l}{(2\pi)^2} x(\mathbf{l}) x'(\mathbf{l}') F(\mathbf{l},\mathbf{l}')$$

X = T, E, B

$\mathbf{d} = \nabla\phi$

1. $< d(\mathbf{L}) > \ = \ L\phi(\mathbf{L})$ **2-point correlation <xx>**

constrain $A(L)$ → **noise**

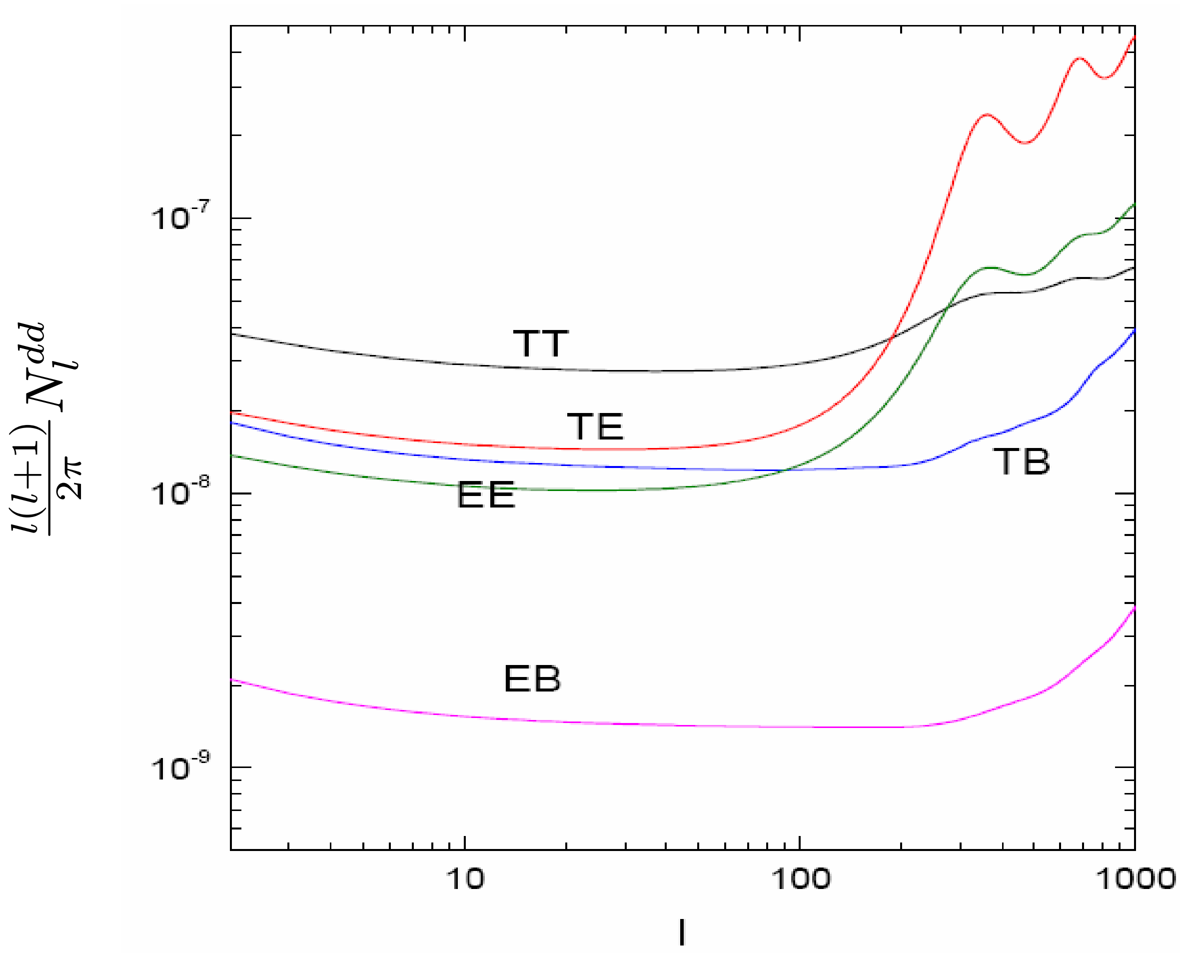
2. $< d^2(\mathbf{L}) > \ = \ L^2\phi^2(\mathbf{L}) + N(L)$ **4-point correlation <xxxx>**

Minimize: $N(L)$

constrain $F(\mathbf{l},\mathbf{l}')$ → **Filter**

F(l,l') contains unlensed (!) and lensed power spectrum

Noise power spectrum for the reference experiment



Convergence for $L_{max} = 3000$

$$\Delta_T = 1\mu\text{K}\cdot\text{arcmin},\ \Delta_P = \sqrt{2}\mu\text{K}\cdot\text{arcmin},\ \sigma = 4'$$

Hu & Okamoto's EB estimator

$$d_{EB}(\mathbf{L}) = \frac{A(L)}{L}\int \frac{d^2 l}{(2\pi)^2} E(\mathbf{l}) B(\mathbf{l}') \frac{\tilde{C}_l^{EE}\mathbf{L}\cdot\mathbf{l}}{C_l^{EE} C_{l'}^{BB}} \sin 2\phi_{\mathbf{l}_1\mathbf{l}_2}$$

$\text{CPU Time} \propto N^2$ ***Very slow***

Define tensor maps (H & O):

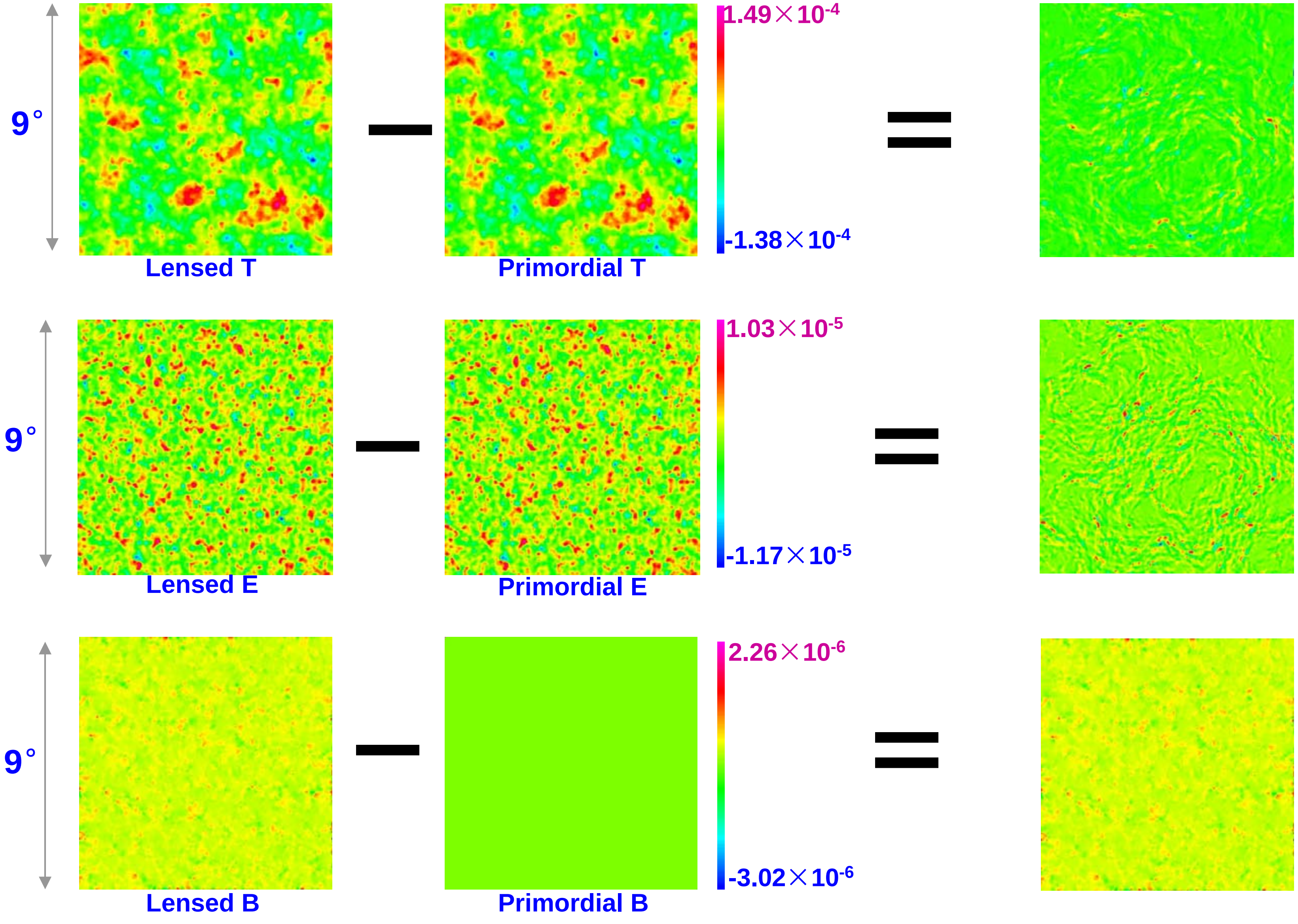
$$M^{(1)}_{ikj} = \int \frac{d^2 l}{(2\pi)^2} e^{i\mathbf{l}\cdot\mathbf{n}} l \hat{l}_i \hat{l}_k \hat{l}_j E(\mathbf{l}) \frac{\tilde{C}_l^{EE}}{C_l^{EE}}$$

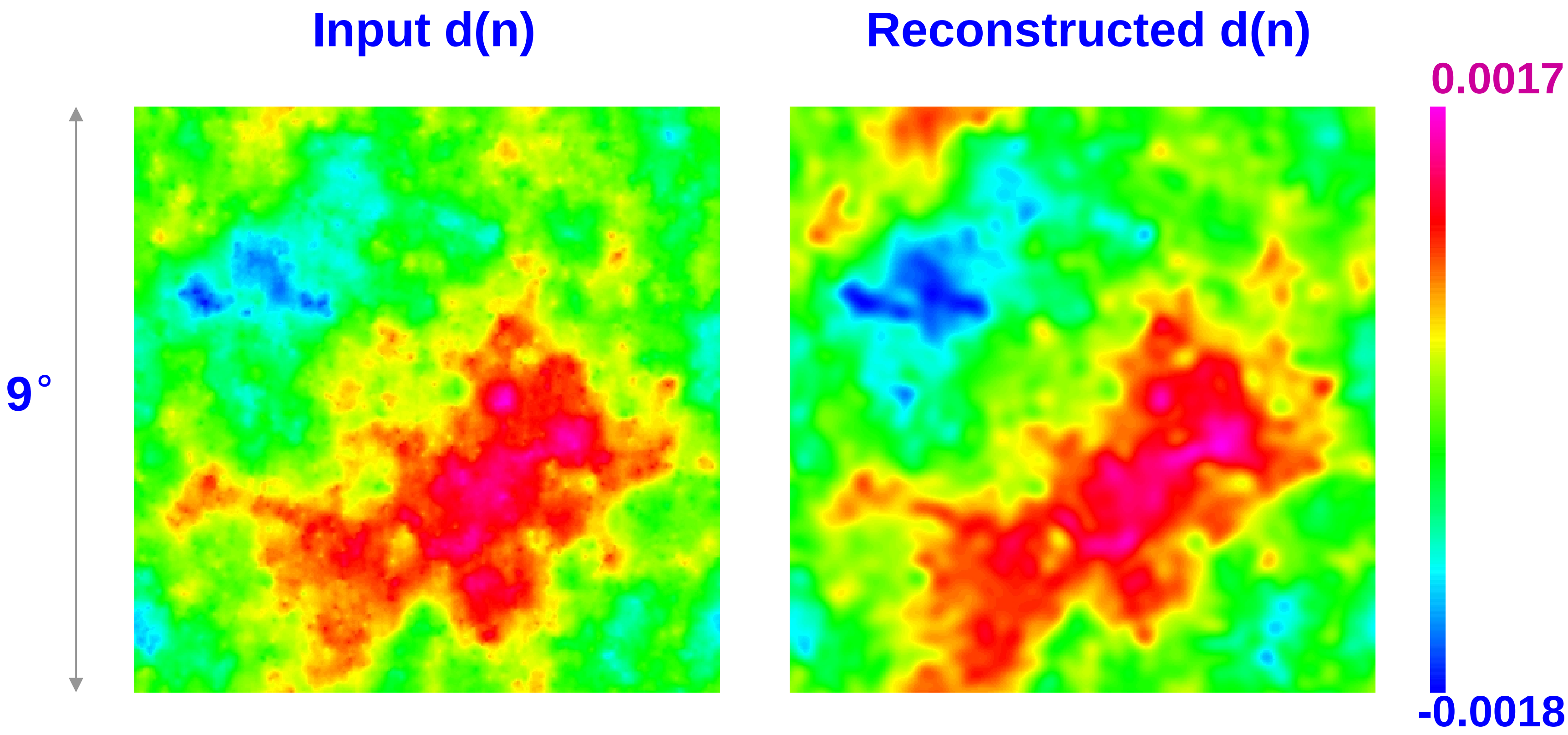
$$M^{(2)}_{km} = \int \frac{d^2 l'}{(2\pi)^2} e^{i\mathbf{l}'\cdot\mathbf{n}} B(\mathbf{l}') \frac{1}{C_{l'}^{BB}} \hat{l}'_k \hat{l}'_m$$

$$d_{EB}(\mathbf{L}) = \frac{A(L)}{L} L_i \mathcal{F}^{-1}[(-2)\sum_{jkm} M^{(1)}_{ijk}(\mathbf{n}) M^{(2)}_{km}(\mathbf{n}) \epsilon_{km3}]$$

$\text{CPU Time} \propto N \log N$ ***Very fast***

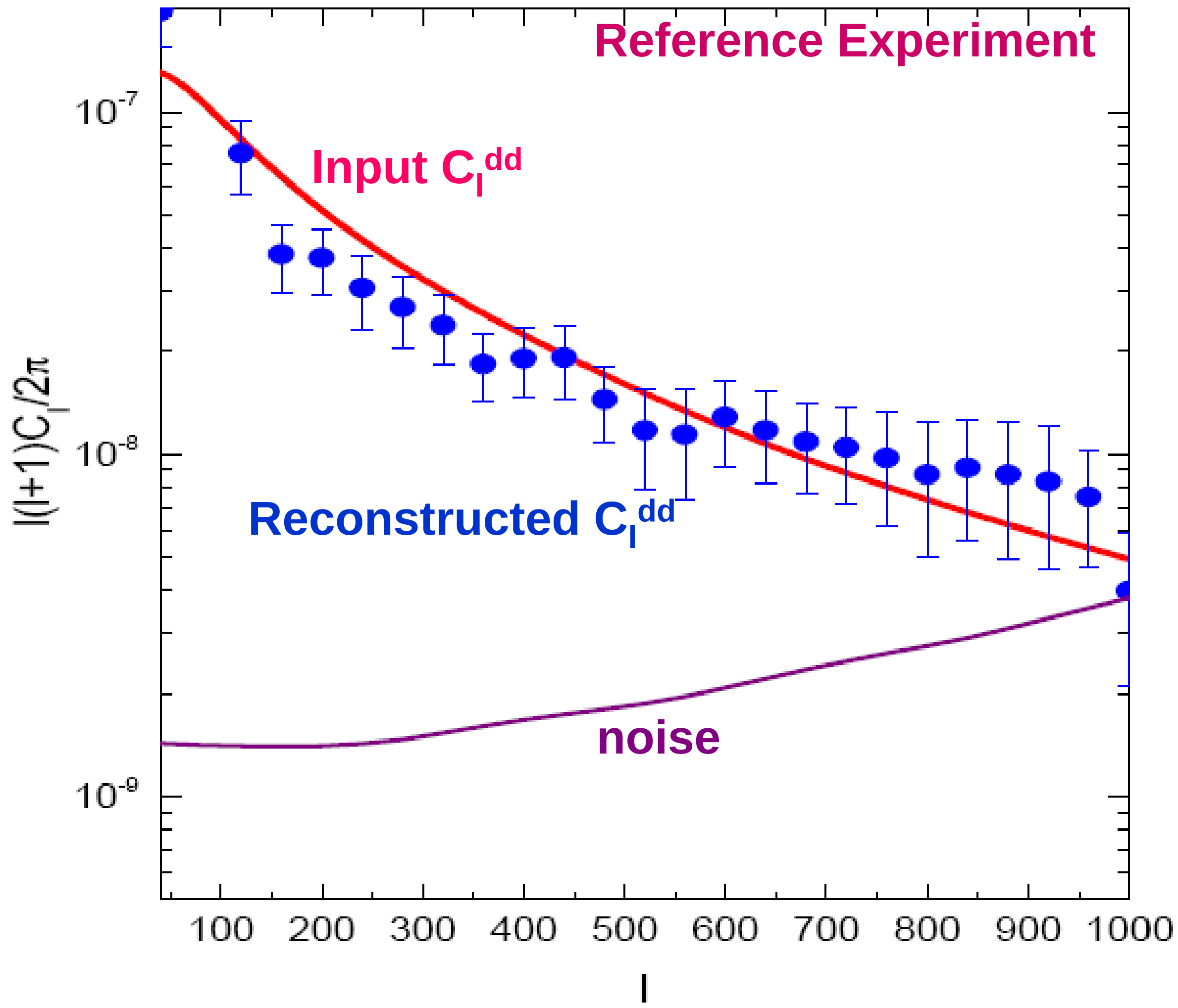
Primordial and Lensed CMB





$$\mathrm{d}(\mathbf{n}) = \int \frac{d^2 l}{(2\pi)^2} e^{i\mathbf{l}\cdot\mathbf{n}} l \phi(\mathbf{l})$$

Averaged reconstructed power spectrum C_l^{dd}



Δ L=40, Lmax=8000, 50 realizations

Outline

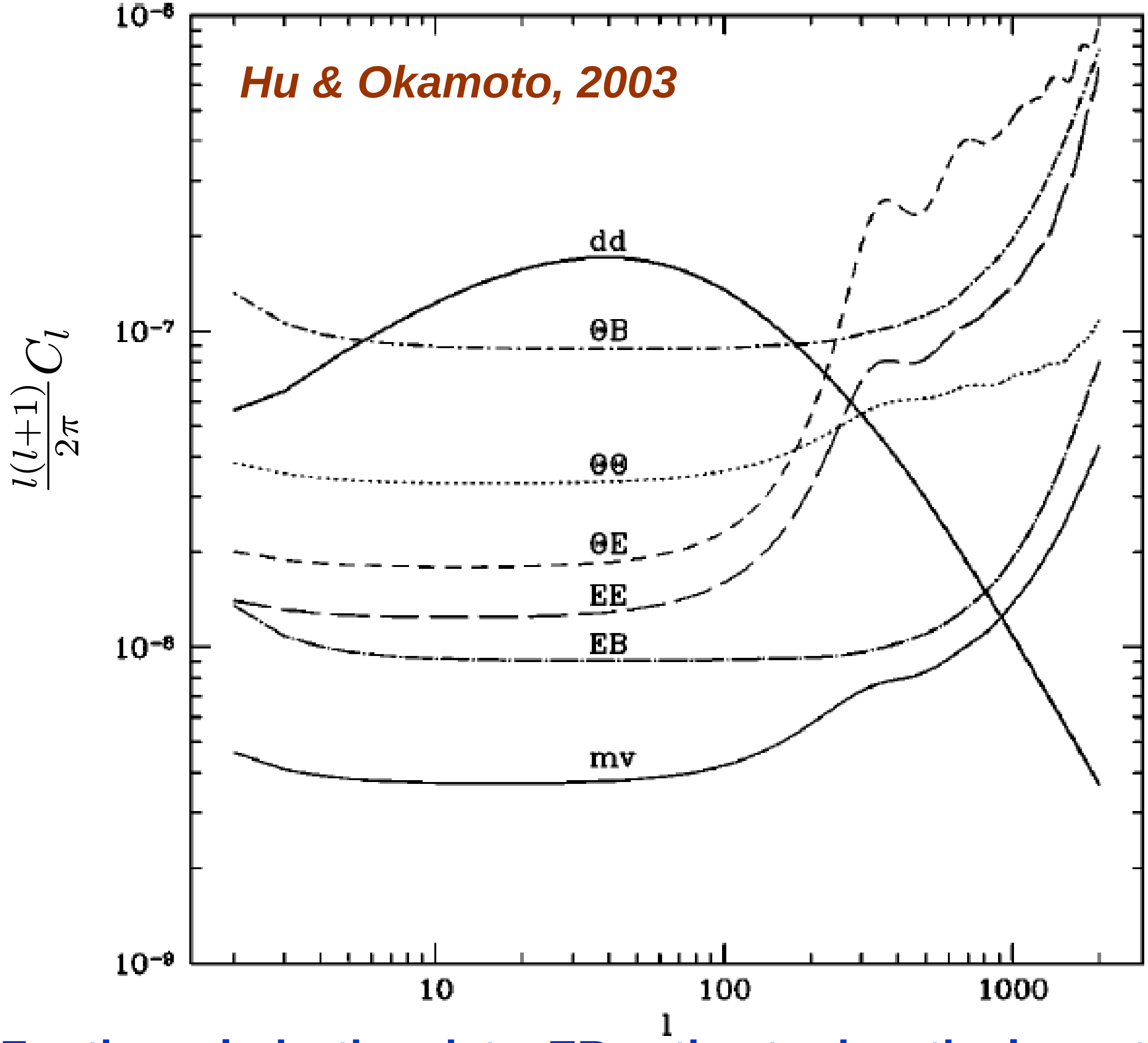
Introduction

Flat & Full sky reconstructions (Hu & Okamoto)

Flat sky reconstruction (Novel Method)

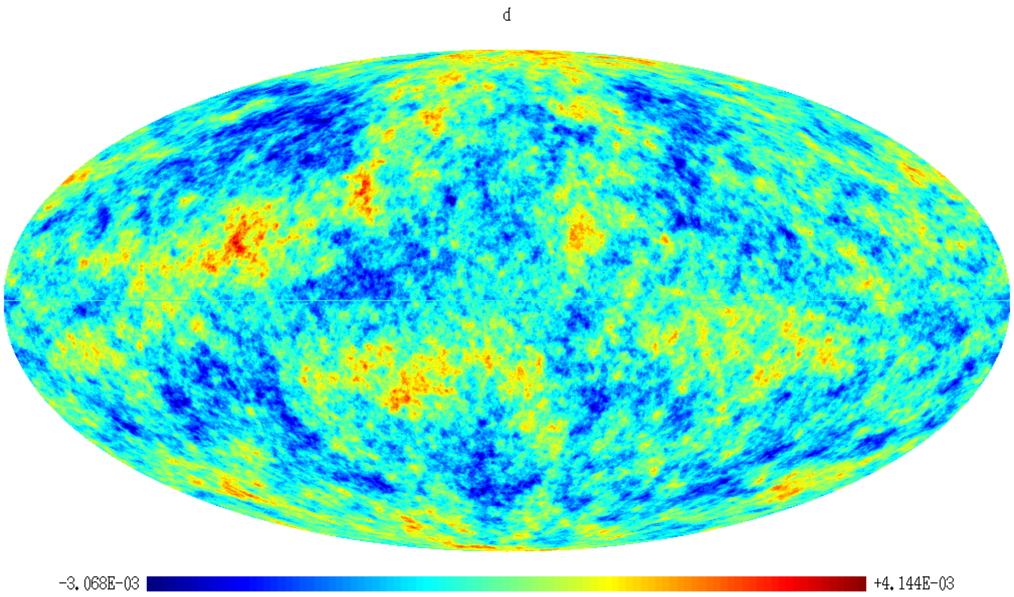
Plan for the future

Signal and Noise in Full Sky

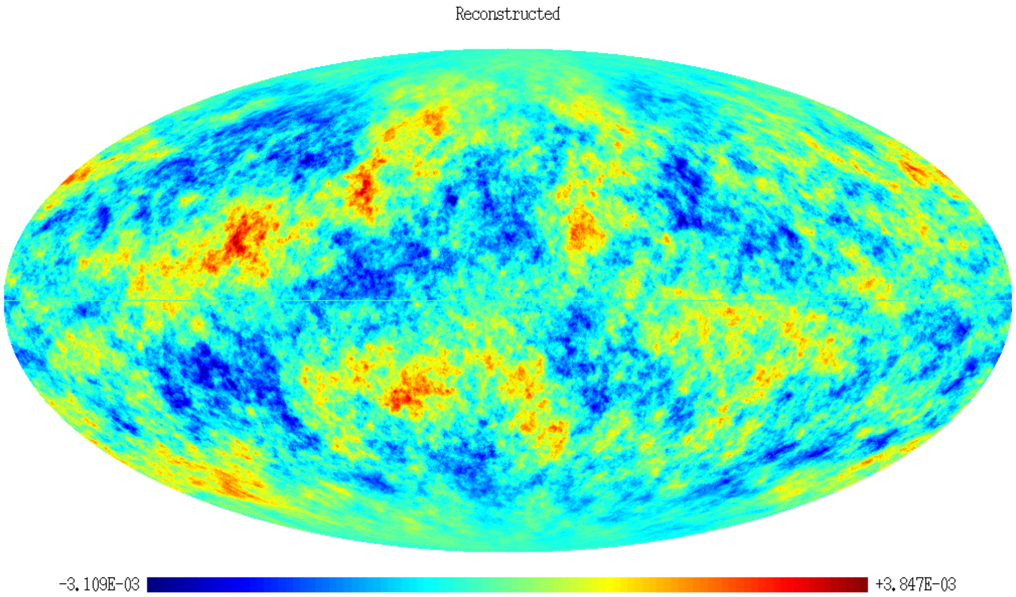


For the polarization data, EB estimator has the lowest noise

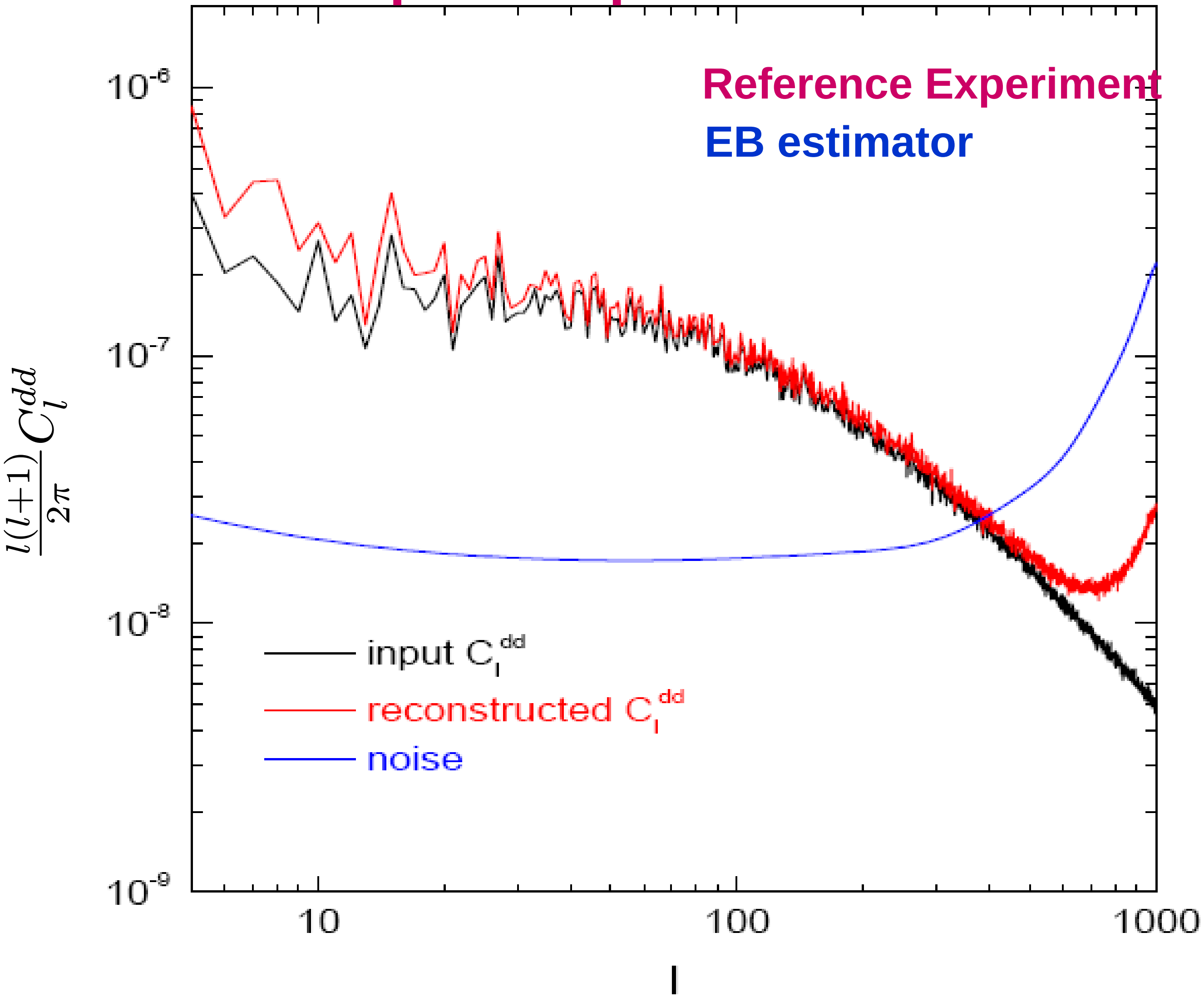
Input d(n)



Reconstructed d(n)



Reconstructed power spectrum



Healpix Pixelization, Nside = 512, Lmax = 1024

$$\Delta_T = 1\mu\text{K}\cdot\text{arcmin},\ \Delta_P = \sqrt{2}\mu\text{K}\cdot\text{arcmin},\ \sigma = 4'$$

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Pixel based optimization

$$\chi^2(\mathbf{d}_i) = \sum_k [\tilde{X}^{(k)}(\mathbf{n}_i + \mathbf{d}_i) - X^{(k)}(\mathbf{n}_i)]^2$$

Simple idea: We simulate the lensed pixel by adding lensing signal corresponding to this pixel, and compare it to the observed pixel. The difference then forces us to get the input lensing signal at this site.

Lensing simulated by convolution

$$X^{(k)}(\mathbf{n}_i) = \tilde{X}^{(k)}(\mathbf{n}_i + \mathbf{d}_i) = \sum_{j=-\infty}^{+\infty} \tilde{X}^{(k)}(\mathbf{n}_j) r(\mathbf{n}_i + \mathbf{d}_i - \mathbf{n}_j)$$

r(n) is the cubic spline basis

S. K. Park, R. A. Schowngerdt.
Computer Graphics Image Processing, 23(1983), 258

Minimization strategy I: Searching in 2 dimensions

$$(n_x + d_x, n_y + d_y)$$

Genetic Algorithm (GA)

↓

Monte Carlo Markov Chain (MCMC)

↓

Broyden-Fletcher-Goldfarb-Shanno (BFGS)

↓

Gradient Descent

O(10^0s) per pixel on the sky

Minimization strategy II: Searching in 1 dimension

$$\left(n_x + \frac{|d|}{\sqrt{2}}, n_y + \frac{|d|}{\sqrt{2}}\right)$$

Search along a line

↓

Gradient Descent

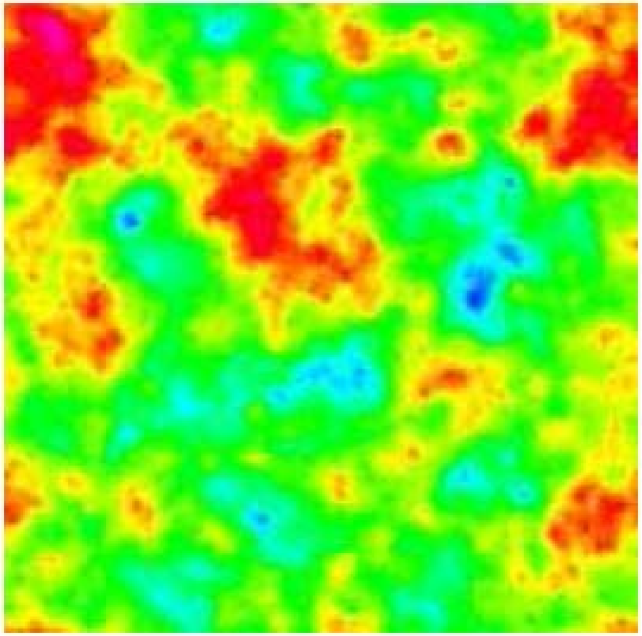
$O(10^{-1}s)$ per pixel on the sky

It is 10 times faster than Strategy I with the same results.

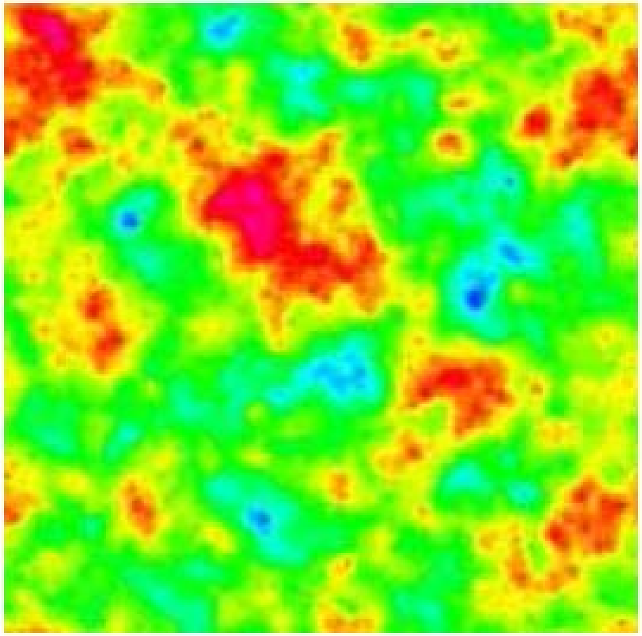
Results

No noise for now

Input d(n)



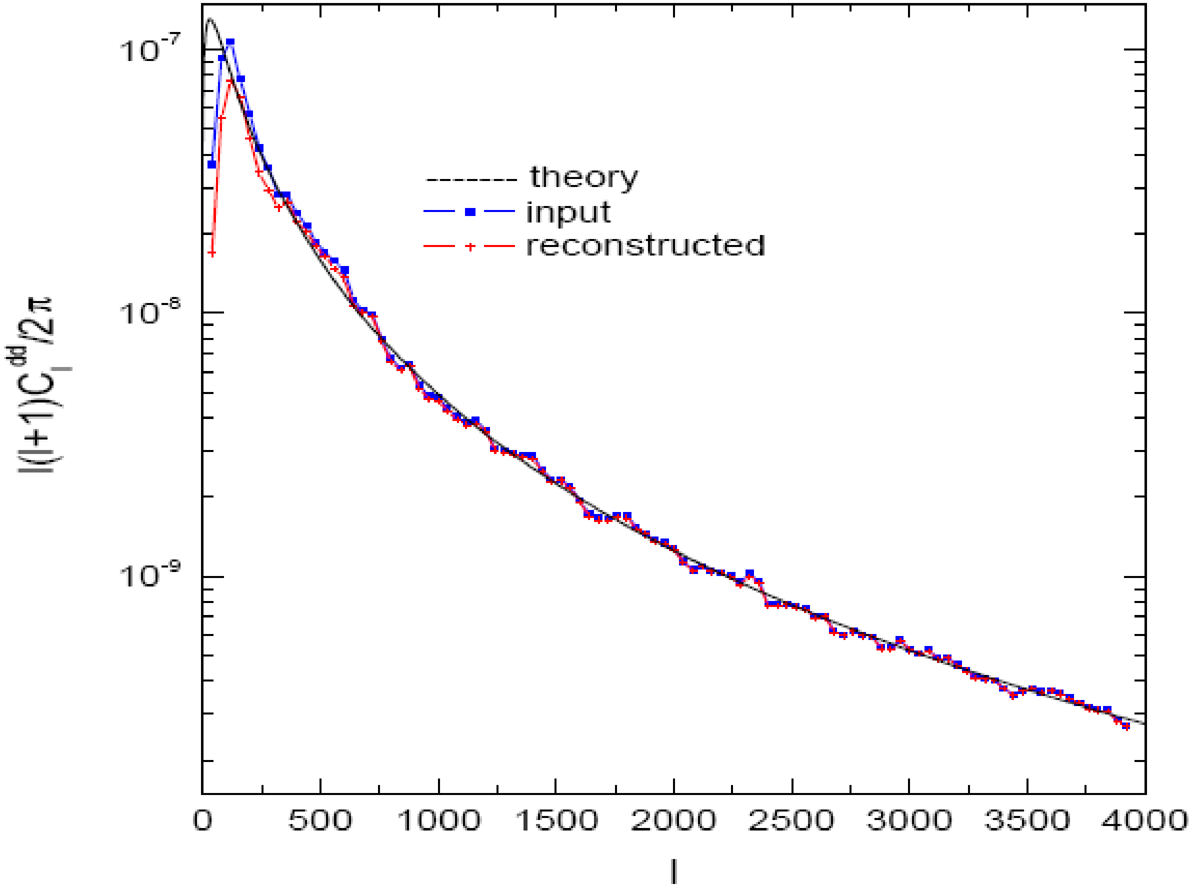
Reconstructed d(n)



$$\Delta_T = 0\mu\mathrm{K{\cdot}arcmin},\ \Delta_P = 0\mu\mathrm{K{\cdot}arcmin}$$

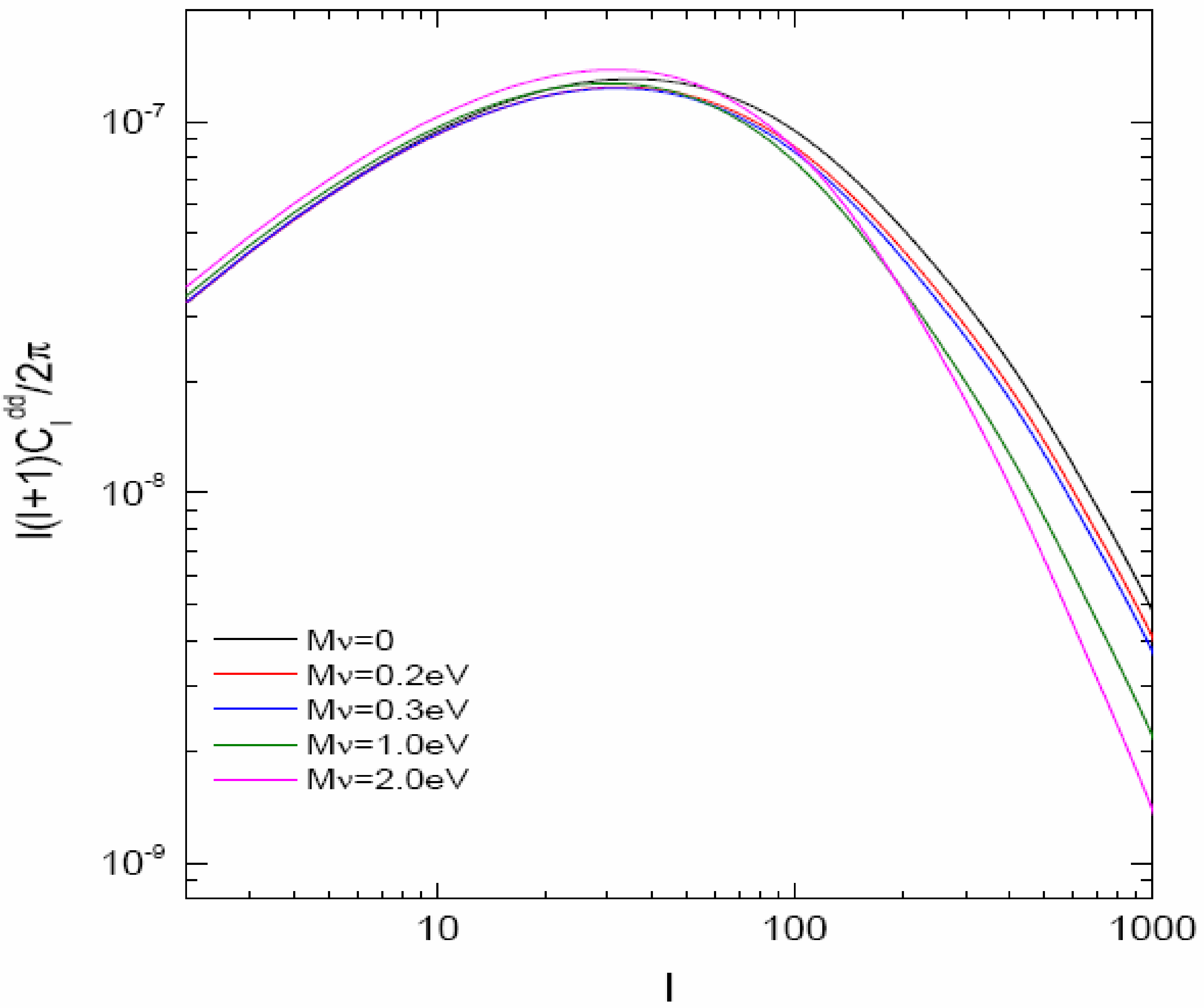
Power spectra

no noise



$$\Delta_T = 0\mu\text{K}\cdot\text{arcmin},\ \Delta_P = 0\mu\text{K}\cdot\text{arcmin}$$

Changes made by neutrino masses

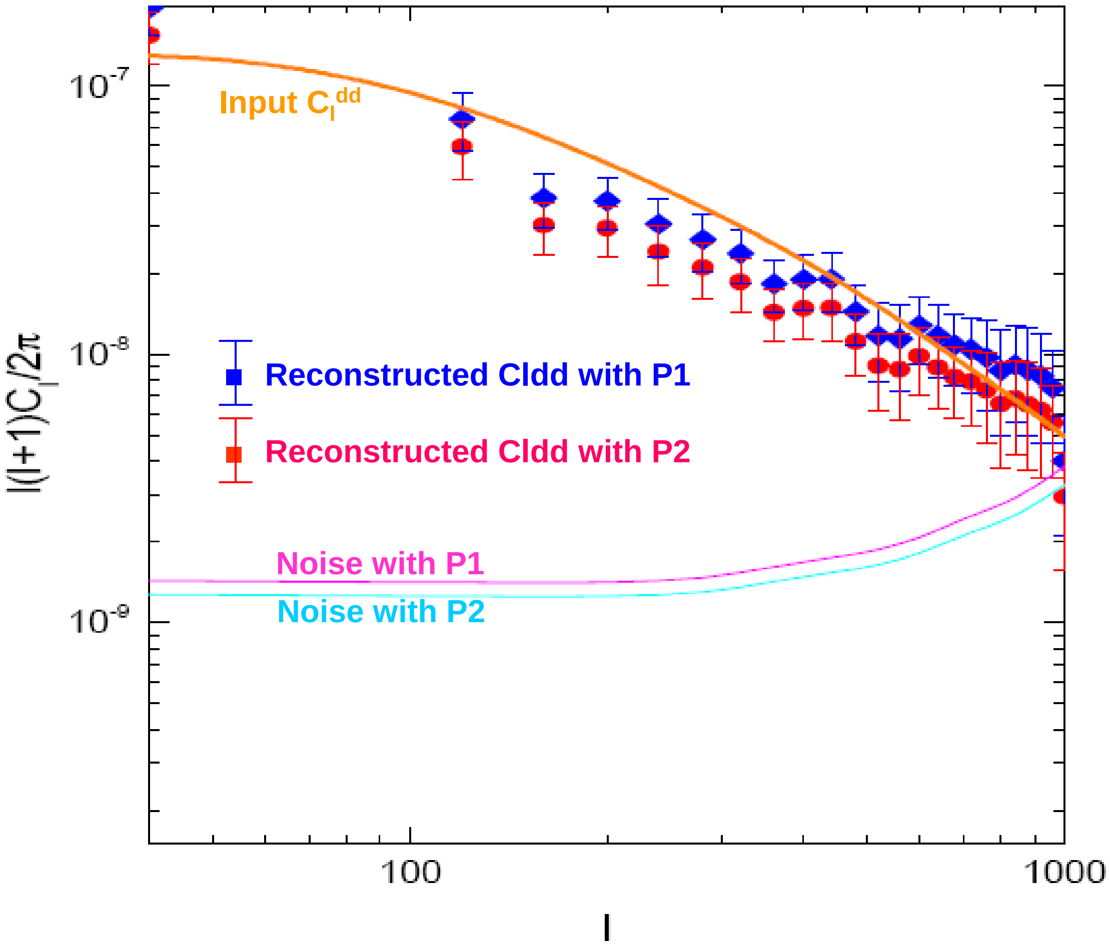


Sensitivity to primordial power spectra for H & O

CAMB default **WMAP 7**

case	P_1	P_2	percent change
$\Omega_b h^2$	0.0226	0.0258	14.2%
$\Omega_c h^2$	0.112	0.1109	0.98%
H_0	70	71	1.43%
n_s	0.96	0.963	0.31%
A_s	2.1×10^{-9}	2.43×10^{-9}	15.7%
τ	0.09	0.088	2.22%

Log scale



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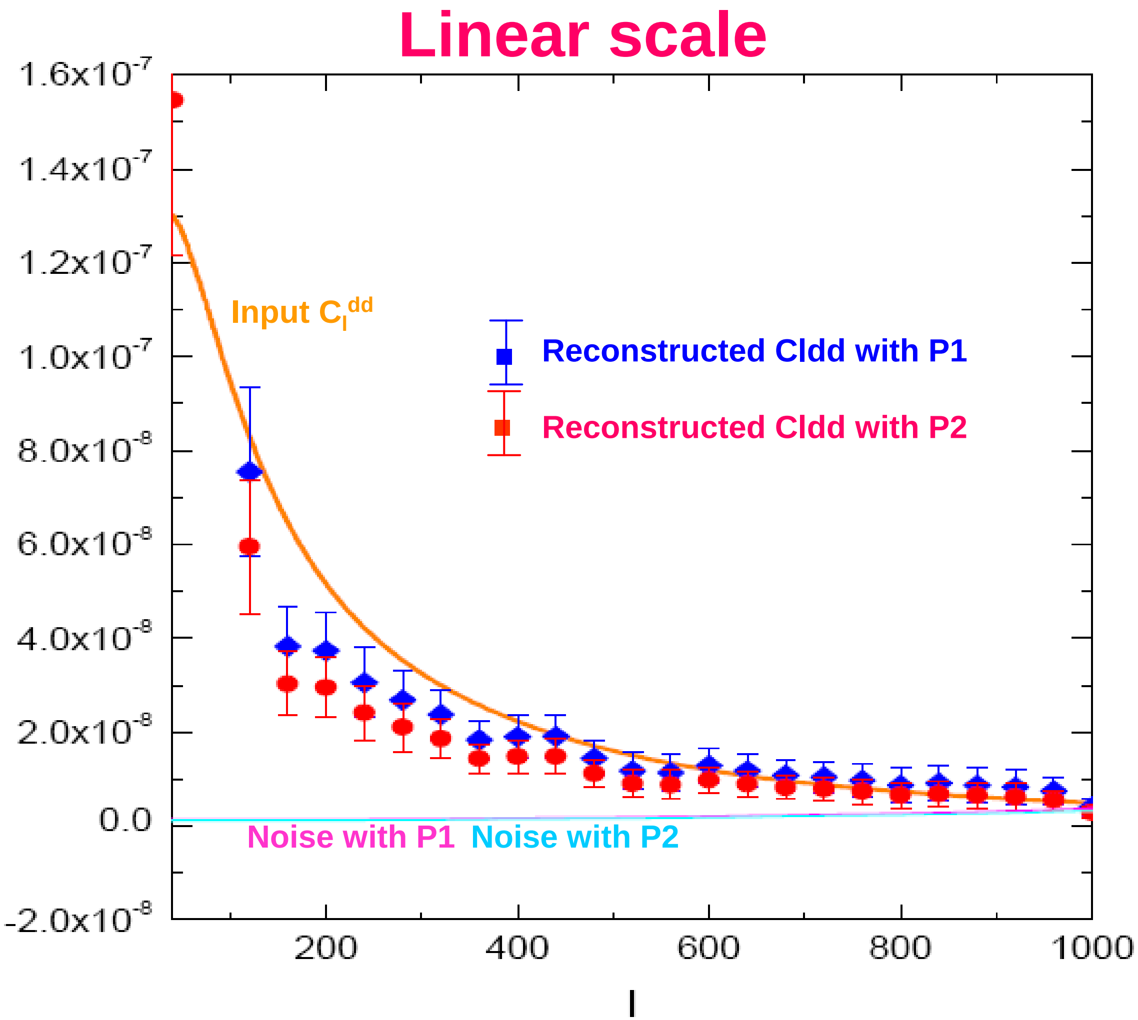

Future plans

- Add instrumental noise to χ^2 method
- Study sensitivity to knowledge of $\tilde{X}(n)$
- Compare Hu & Okamoto method to χ^2 method
- Forecast for upcoming experiments (Planck, CMBPol, Polarbear, ACTPol, SPTPol)

Conclusion

De-lensing involves lensed and primordial power spectra

Extraction of deflection and primordial B mode power spectra will be a challenge

We are probably reaching the point where theoretical systematic effects will become significant

Thank you!